The (Un)Sustainability of the U.S. International Debt Position:

Do Not Rely Too Much on Valuation Effects

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Abstract

In this paper, I claim that there are sound theoretical and empirical reasons to believe that the valuation channel is unimportant for the sustainability of the American international debt, in spite of the role it plays in the adjustment dynamics of the external imbalances. First, I observe that the theoretical relevance of the valuation channel does not pertain to stochastic equilibrium models; it emerges only in deterministic or linearized setups. Second, I provide empirical evidence that the U.S. debt has displayed features of unsustainability for the period 1989-2004. Then, I use a Bayesian VAR to confirm the different theoretical predictions of stochastic and linearized models. My results restore the traditional primary role of the trade channel for sustainability.

JEL: F31, F32, F34, F37

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The large and persistent U.S. current account deficit of the last two decades has raised many concerns about the sustainability of the American international net debt position. As any other borrower, also the U.S. is expected to generate positive savings at some point in the future in order to meet its current liabilities. On the one hand, the standard conclusions of the intertemporal approach of the current account imply that, soon or later, it will have to invert its trade balance and become a net exporter. On the other hand, the valuation channel theory, recently introduced by Pierre-Olivier Gourinchas and Helene Rey (2007b), has complemented the conventional intertemporal approach by pointing out the systematically corrective role of the valuation effects on the American international portfolio positions in the adjustment process of the U.S. external imbalances in deviations from slow-moving trends between 1952 and 2004. This new channel

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cooperates with the trade channel in eliminating the external imbalances and it is particularly active at the shorter horizons.

This paper takes a thorough look at the actual relationship between valuation effects and sustainability of the foreign debt. In spite of an undeniable impact of the valuation effects on the accumulation process of debt, I provide both theoretical considerations and empirical evidence for the U.S. that completely downplay the role of the valuation channel in achieving sustainability.

Maurice Obstfeld and Kenneth Rogoff (2000 and 2007) and Olivier Blanchard, Francesco Giavazzi, and Filipa Sa (2005) have provided the most prominent examples of the earliest literature on the sustainability of the American current account that focuses on the rebalancing mechanisms expressed by the trade channel. In these models, a large real depreciation of the dollar triggers the shift of the international demand in real markets and the switch of sign of the trade balance necessary to drive the adjustment process of the current account.

The depreciation of the dollar is the common denominator of this traditional channel and the valuation channel because a depreciation of the home currency induces also positive capital gains on the foreign assets held by a country and denominated in other foreign currencies, which alleviates the burden on the future trade surpluses. For this reason, the valuation channel has caught the attention of economists and policy makers, in particular in this global economy characterized by large international imbalances and the increasing holdings of international assets in many countries (as documented in Philip R. Lane and Gian Maria Milesi-Ferreti (2004 and 2007)).

The role of the valuation channel of Gourinchas and Rey can be interpreted as a parallel to that of the monetary policy in a traditional non-Ricardian regime. The results in Gourinchas and Rey (2007b) suggest that the valuation effects have been restraining the growth of the U.S. international debt in past decades, in face of insufficient trade balances, in the same way as seigniorage would compensate a non-Ricardian fiscal policy in order to guarantee the sustainability of government debt.

This paper makes the crucial point that these conclusions, drawn for the rebalancing dynamics of the net foreign debt, cannot be safely extended to the sustainability of the debt because Gourinchas and Rey study only the fluctuations around slow-moving trends of the variables of interests after imposing by assumption the sustainability itself.

I start from the clear theoretical predictions of a stochastic general equilibrium model for the economy-wise budget constraint in a standard two-country open economy framework, in which the sustainability of the net foreign asset position coincides with the transversality condition on the intertemporal version of the

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1I follow here the distinction between *traditional* Ricardian and Ricardian regimes used, for example, by Carl Walsh (2004) Ch. 4 to distinguish between the standard concept of Ricardian fiscal policy and the concept of Ricardian regimes in the fiscal theory of price level literature.
budget constraint being satisfied. In this setup, returns are optimally determined in relation to the stochastic consumption-based discount factor of the representative household and are part of the recursive definition of assets and liabilities in the budget constraint.

The stochastic discount factor of the household is the correct discount factor to be considered in the transversality condition then, and not, as it is often done, the net return on the debt position. Assuming that the net return is a risk-free rate and using it as the relevant intertemporal discount factor is acceptable only in a framework in which risk does not play any role, as, for example, when preferences are risk neutral or in a deterministic model. The linearized version of a stochastic model deals with risk in the same way and can entail a predictability of the excess returns through the budget constraint. On the contrary, once risk is correctly taken into account by the model, this predictability is not theoretically justified any more.

With these theoretical premises in mind, the empirical goal of the paper is to assess the issue of the sustainability of the American foreign debt position in light of the previous evidence on the role of the valuation effects. Two main points are of concern. The first is whether the international net debt position of the U.S. has been on a sustainable path since it started growing at a faster pace in the early 90s. The second point is checking whether the predictions of the stochastic model on the role of the valuation channel and the predictability of the excess returns are empirically confirmed.

My analysis casts serious doubts on the sustainability of the trajectory of the U.S. net foreign debt over the period 1989-2004, which is the time window in Gourinchas and Rey’s data set in which the U.S. become a net borrower. I do not find predictability of the excess returns using data in levels, which implies that the entire sustainability rests on the expectations of positive future trade balances. I interpret this as evidence against non-Ricardian dominance of the trade deficit over the returns, even though the trade balances have not been vigorous enough to keep the debt on a sustainable trajectory.

The empirical strategy I follow is organized in two steps too. First, I evaluate the expected value of the limit in the transversality condition exploiting the sample properties of the stochastic discount factor and the debt series and I find that the realized observations seem to violate the zero limit condition.

Second, I estimate a four-variables Bayesian VAR in levels that includes the trade surplus, the net international debt, the foreign asset excess returns, and the depreciation rate of the real exchange rate. The impulse response functions of the trade surplus and the excess returns to a shock to the net debt suggest that the sustainability is entirely based on the future improvements of the trade balance. The excess returns respond on impact to the shock, but then the responses are not significant anymore. I compare these results to those obtained from a de-trended version of the same VAR, treating the data series as in Gourinchas and Rey (2007b). This version of the VAR is meant to be an empirical counterpart of the theoretical linearized model. Interestingly and in line with the empirical evidence in Gourinchas and Rey, the different
The predictability of the excess returns implied by the linearized model is confirmed by this second type of VAR. Thus, a useful stabilization mechanism is revealed, which, however, is not sufficient to imply sustainability.

Given the centrality of the exchange rate in the adjustment mechanisms of both the channels, the last section of the paper makes an attempt to provide a further insight about the endogenous determination of the real exchange rate in the context of the VAR model used in this paper. The same approach adopted by Matthew B. Canzoneri, Robert E. Cumby, and Behzad Diba (2001) to test the fiscal theory of the price level can be applied to the analysis of the relation among foreign debt, trade balances, and real exchange rate.

The responses of the net debt to shocks to the trade surplus could unfold a non-Ricardian dynamics of the real exchange rate. In this case, the real exchange rate, which plays the same role as the domestic price level in the fiscal theory of the price level, might jump to restore the equilibrium in the debt position after an unexpected increase of the trade surplus. The real exchange rate should appreciate in order to generate lower returns on foreign assets and the net debt should rise to match the increase in the trade surplus. I find that the exchange rate actually depreciates and the asset returns move in the wrong direction, showing that the behavior of the real exchange rate is more consistent with a Ricardian regime.

The results of this last section should be regarded as a corollary of the broader discussion undertaken in this paper. Empirical evidence in this respect must always be read very cautiously since it is strongly conditioned by the empirical ambiguity of the theoretical concept of off-equilibrium variables. Here, the separation between foreign assets and liabilities and the fact that the real exchange rate is only one of the factors affecting asset prices make the interpretation of these results even more difficult.

0.0.1 Relation to the Literature

There exists a vast amount of literature on testing government debt sustainability. I briefly review it in the next section, adapting it to the case of international debt with significant valuation effects. In the earlier literature, for example, Marjorie A. Flavin and James D. Hamilton (1986), David W. Wilcox (1989), and Bharat Trehan and Carl E. Walsh (1988) proposed tests based on the maintained assumption that the debt position is stationary or integrated of order one, but these tests are not fully satisfactory because they simply impose that the transversality condition is fulfilled.

The only necessary and sufficient condition required for sustainability is that debt grows at a smaller rate than the relevant discount factor so that the limit of the transversality condition converges to zero in expectations. However, since showing the convergence of this limit is not a straightforward task, a second strand of literature has focused on providing and testing simpler sufficient conditions for sustainability. Trehan and Walsh (1991), Merih Uctum and Michael R. Wickens (1993), and Henning Bohn (1998) in particular, check for endogenous responses of the budget surplus to the level of debt that automatically
fulfill the transversality condition.

Most of the literature has used deterministic discount factors in studying the transversality condition. In a stochastic environment, however, the choice of the discount factor cannot be arbitrary because it has to be endogenously determined by the model itself. As pointed out by Bohn (1995 and 1998), it is theoretically correct to derive the intertemporal budget constraint and the transversality condition using a consumption-based stochastic discount factor obtained from the optimality conditions of the representative household’s problem. I follow this insight in my empirical strategy to evaluate the transversality condition.

Even though this paper might seem antithetical to Gourinchas and Rey (2007b) at first sight, its correct positioning would be next to theirs. Gourinchas and Rey impose the transversality condition, hence the sustainability of the net debt position, and then explore some theoretical relations between the net foreign asset position, future trade balances, and future asset returns. In this paper, I adopt a stochastic version of the same kind of setup, but I proceed by studying the transversality condition rather than imposing ex-ante the sustainability. For the same reason, this work is complementary to the literature that describes the current imbalances as the outcome of equilibrium models. Although they pursue extremely different perspectives on the problem, all the works in this area, from Ricardo Caballero, Emmanuel Farhi, and Gourinchas (2008) to Nicolas Coeurdacier, Robert Kollmann, and Philippe Martin (2010), draw their conclusions conditionally on the imposition of the sustainability of the debt.

Canzoneri, Cumby, and Diba use a two-variable VAR for the government debt and the budget surplus to find whether the fiscal theory of price level holds for the U.S. I use a VAR too as an analytical tool to study the relations among the variables of this model. As in Canzoneri, Cumby, and Diba, one can make some considerations on possible non-Ricardian regimes of the real exchange rate looking at the responses to the trade balance shocks. The main purpose here, however, is to use the responses to the debt shocks in order to elucidate the mechanisms that support the sustainability of the debt.

The rest of the paper is organized as follows: In the next section, I introduce the concept of sustainability implied by the intertemporal budget constraint and I review some popular tests of sustainability in the context of international debt. Section 2 presents the open economy stochastic general equilibrium model I use as a theoretical reference for the derivation of the transversality condition. Section 3 is about the data for the empirical part. In Sections 4–6, I explain and implement the empirical strategy and I present the results. Section 7 concludes.

\(^2\)They impose a well-defined, balanced growth path and the stationarity of the net assets to wealth ratio around this deterministic trend. This is the typical assumption in growth models and it makes the transversality condition satisfied by construction (see Bohn (2008)).
1 Testing for Sustainability

1.1 Implications from The Budget Constraint

The common textbook definition of current account assumes that the same rate of return is paid on foreign assets and liabilities and, disregarding any valuation effect, links the current account deficits to the accumulation of net foreign debt. Equation (1) defines the ratio to GDP current account $ca_t$ as

$$b_{t+1} - b_t = -ca_{t+1} = (r_{t+1} - g_{t+1})b_t - d_{t+1}$$

(1)

where $b_t$ is the ratio of the net foreign liabilities to nominal GDP at the end of period $t$, which can be decomposed into foreign assets $a_t$ and liabilities $l_t$ so that $b_t = l_t - a_t$. The nominal (ex-post) rate of return on the stock of net debt realized between period $t$ and period $t+1$ is $r_{t+1}$; $g_{t+1}$ is the growth rate of nominal GDP between period $t$ and period $t+1$; finally, $d_{t+1}$ is the trade balance as a ratio to GDP.

Solving (1) forward for $b_t$, and assuming for sake of simplicity a constant rate of return $r$ and GDP growth rate $g$, gives the following well-known expression of the deterministic intertemporal budget constraint

$$b_t = \sum_{i=1}^{\infty} \left( \frac{d_{t+i}}{(1+r-g)^i} + \lim_{T \to \infty} \frac{b_T}{(1+r-g)^{T-t}} \right)$$

(2)

For the intertemporal budget constraint to have full economic meaning, the discount rate $r$ must be greater than $g$ and the transversality condition (3) must hold

$$\lim_{T \to \infty} \frac{b_T}{(1+r-g)^{T-t}} = 0$$

(3)

Under this condition, we can say that the intertemporal budget constraint is fulfilled. This is the concept of sustainability of the net foreign debt I am going to use in this paper, which corresponds, in practice, to testing whether the transversality condition holds in the data.

If, for example, one assumes that $\Delta b_t$ (or, equivalently, $ca_t$) is stationary, then the transversality condition is easily satisfied. $b_t$ would be at most an $I(1)$ process and the exponential growth rate of the discount factor in (3) would rule out an excessively rapid accumulation of net foreign liabilities, as shown by Bohn (2008). A prolonged negative current account position is also consistent with this hypothetical case, but the mean reversion property of $\Delta b_t$ makes a rebalancing of the current account certain in finite time. However, this

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3The exact equation corresponding to (1) is

$$b_{t+1} = \frac{1 + r_{t+1}}{1 + g_{t+1}} b_t - d_{t+1}$$

Equation (1) is an approximated version that makes the following derivations simpler.
kind of consideration is appropriate only if $\Delta b_t$ is assumed to be stationary, which is not necessarily the case, and if $ca_t$ in (1) can be considered a theoretically correct counterpart of $\Delta b_t$.

For example, Flavin and Hamilton, Wilcox, and Trehan and Walsh propose tests for sustainability based on the maintained assumption that $b_t$ is either stationary or integrated of order one. They also arbitrarily take the return rate paid on the net debt as discount factor in the intertemporal budget constraint. This approach is not correct from a theoretical point of view, because it is equivalent to impose a-priori the sustainability of the process, which is the final object of the test. In the specific case of the American net foreign debt, this is not correct from an empirical point of view either. There are three reasons why this kind of approach cannot be considered satisfactory.

The first is that valuation effects on the gross positions have been shown to be quite large for the U.S. (see Gourinchas and Rey (2007a)). There is a crucial difference between the financial flow of net foreign liabilities and the actual change of the net asset position $\Delta b_t$. The accounting definition of the current account measures the financial flow rather than $\Delta b_t$. The stationarity of $\Delta b_t$ does not necessarily imply anymore the stationarity of the current account if $b_t$ is non-stationary. For this reason, considerations of the required rebalancing of the current account should be based on the sustainability of the intertemporal budget constraint and not on the size of the current account.

The second reason is that the rates of return on foreign assets and foreign liabilities have differed in a systematic way in the past decades, with Americans receiving a higher return on foreign assets than they paid on liabilities. This is the exorbitant privilege reported by Gourinchas and Rey (2007a). Stephanie E. Curcuru, Tomas Dvorak, and Francis E. Warnock (2008) dispute the actual size and importance of these returns differentials. However, it will be clear from the following analysis that the privilege does not matter for sustainability.

The third reason is that the application of a constant discount factor derived from the net return rate in the budget constrain is limited to models in which either the agents are risk neutral or there is perfect foresight, even when the valuation effects or the double returns problem is neglected. In these cases, if the debt issued by a country is assumed to pay a risk-free return, the discount factor is anchored to the intertemporal preferences coefficient of the lending agent by the Euler equation.

1.2 Endogenous Responses of the Trade Surplus

Although stabilization does not necessarily imply sustainability, since it may just cover a potential long-run explosive behavior of the debt, some stabilization mechanisms also provide sufficient conditions for sustainability to hold.

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*Such a condition would be actually a sufficient condition to satisfy the intertemporal budget constraint.*
A good test for sustainability would be to check for the existence of systematic responses of the trade surplus (or the valuation effects) to the level of debt that allow the transversality condition to be satisfied. These are only sufficient conditions, however, not necessary. The only necessary and sufficient condition for the transversality condition to hold is that the net debt grows (in expectations) at a rate lower than that at which the discount factor shrinks.

It might seem more convenient to directly test for the transversality condition once an appropriate stochastic discount factor has been defined, but this actually requires being able to infer the distribution of the debt over the full set of states of nature from a single observation of data in order to evaluate the expectations of future discounted debt. On the contrary, testing for endogenous responses is safer and avoids these difficulties.

The intuition supporting this kind of test can be easily seen in the basic case with perfectly substitutable assets and no valuation effects. The equation representing the dynamic of the net debt is the same as in (1). Assume for a moment that the return rate and the GDP growth rate are constant and given by \( r \) and \( g \) with \( r > g \) and that the relevant discount factor for the transversality condition is given by \( 1 + r - g \).\(^5\) We can rewrite (1) using a polynomial in the lag operator \( L \) as

\[
[1 - (1 + r - g) L] b_t = -d_t \tag{4}
\]

Assuming now that \( d_t \) endogenously responds to \( b_t \) and taking a simple linear reaction function

\[
d_t = \alpha b_{t-1} + u_t \tag{5}
\]

where \( u_t \) is an exogenous white noise, it is easy to see that the root of the polynomial becomes \([1 + r - g - \alpha]^{-1}\).

Even if the polynomial is not invertible, and \( b_t \) is non-stationary, any positive value of \( \alpha \) would be sufficient to make the growth rate smaller than the discount rate. If \( \alpha \) is positive and \( \alpha > r - g \) then the process would also be invertible, while negative values of \( \alpha \) determine unsustainability.

Estimating \( \alpha \) in a regression of \( d_t \) on \( b_{t-1} \) provides an interesting approach to test for sustainability as proposed first by Trehan and Walsh (1991). The idea of endogenous responses is adopted also by Bohn (1998), in a univariate case, and by Uctum and Wickens (1993) and, indirectly, by Canzoneri, Cumby, and Diba in a VAR analysis of the joint process generating \([d_t, b_t]\) which embeds the reaction mechanism.\(^6\)

\(^5\)This is the case if the economy has to be dynamically efficient. If \( r < g \), it would always be possible to roll the debt over while simultaneously having a contraction of the debt to GDP ratio.

\(^6\)Bohn (1998) shows that the rule he estimates satisfies the transversality condition in a stochastic framework. Uctum and Wickens (1993) and Canzoneri, Cumby, and Diba apply a simpler ad hoc discount rate in their papers. Canzoneri, Cumby, and Diba use the same kind of approach studying the fiscal theory of price level instead of debt sustainability per se, in order to check if endogenous responses of prices to debt have been keeping the US government debt on a sustainable path.
In Section 4.1, I propose a new procedure to statistically evaluate the transversality condition, based on the sample first moment of the product of the stochastic discount factor and the debt series. I also estimate equation (5) to complement my results.

2 A Stochastic Framework for Sustainability Analysis

Following the considerations in Bohn (1995), in this paper I study the sustainability of the U.S. foreign asset position in the context of an open economy stochastic framework. This section traces out only the main features of a basic model that fulfills the task because it would be beyond the scope of the paper to present it in full details.

The reference model is a typical dynamic stochastic general equilibrium (DSGE) model used in the international real business cycle (RBC) literature, for example as in Jordi Gali and Tommaso Monacelli (2008) for the case of a small open economy. Since the analysis hinges on the U.S., it is reasonable to assume here that the world economy is formed by two symmetric countries: the domestic country, which is represented by the U.S. in the data, and rest of the world. Each economy is inhabited by a single representative agent who incorporates the decisions of the households, the production sector, and the government. I assume that the financial markets are internationally complete, which implies that the two economies share the same intertemporal stochastic discount factor.

The rest of the model follows the recent literature on endogenous international portfolio allocation as in Coeurdacier, Kollmann, and Martin, for example, or Michael B. Devereux and Alan Sutherland (2008). The period utility function of the representative household in the home country is defined over consumption and labor supply. It is assumed to be separable in the two arguments and with constant risk aversion. The lifetime utility function is

\[ E_t \sum_{i=0}^{\infty} \beta^{t+i} u(C_{t+i}, N_{t+i}) \]

\[ u(C_t, N_t) = \frac{C_t^{1-\sigma} - N_t^{1+\phi}}{1-\sigma} \]

where \( E_t \) is the expectation operator.

I consider the simplest production structure in open economy with two tradable, imperfectly substitutable goods, and flexible prices. The whole model is treated in real terms only.\(^7\) A TFP shock \( Z_t \) enters the aggregate production function of the domestic firm, which is linear in labor \( Y_t = Z_t N_t \). The firm issues

\(^7\) The introduction of domestic sub-varieties or price stickiness can be done following one of the many examples in the open economy DSGE literature, as in Gali and Monacelli for instance. However, it would not add any particular insight to the main message of this paper.
equity and distributes dividends to the stock holders satisfying its resources constraint. The usual CES aggregator defines the consumption aggregate (6) and its corresponding price index (7)

\[ C_t = \left[ \delta^{\frac{1}{\lambda}} C^{\frac{1}{\lambda}}_{t,H,t} + (1 - \delta) \delta^{\frac{1}{\lambda}} C^{\frac{1}{\lambda}}_{t,F,t} \right]^\frac{1}{1-\lambda} \]  

\[ P_t = \left[ \delta P^{1-\lambda}_{t,H,t} + (1 - \delta) \delta P^{1-\lambda}_{t,F,t} \right]^{\frac{1}{1-\lambda}} \]  

where \( \lambda \) is the elasticity of substitution between home and foreign goods, \( P_{H,t} \) and \( P_{F,t} \) are the price of the domestic and foreign goods in domestic currency, and \( \delta \) is the home bias in consumption, which has the interpretation of steady state share of imports.\(^8\) At each period, the government purchases the amount of composite good \( G_t \) and finances its spending by levying lump-sum taxes and issuing risk-free bonds. It is assumed that \( G_t \) does not directly yield utility.\(^9\)

### 2.1 The Economy-Wise Budget Constraint

A complete set of internationally traded state-contingent securities are available. Let \( \Omega_t \) denote the real payoffs of the Arrow-Debreu securities in term of domestic composite good at time \( t \). The standard budget constraint of the household can be consolidated with the government budget constraint and the resources constraint of the representative firm in order to form the following economy-wise budget constraint

\[ C_t + G_t + E_t \left[ \mu_{t,t+1} \Omega_{t+1} \right] = \Omega_t + P^H_t Y_t \]  

where \( \mu_{t,t+1} \) is the real stochastic discount factor relevant for the domestic agent and \( P^H_t Y_t \) is the real value of domestic production in terms of the composite good.\(^10\)

In the actual world, the payoffs \( \Omega_t \) can be the dividends paid by private equity or the returns on national governments debts and private bonds. Some of these are issued by the U.S., others by the rest of the world. Given the framework of this paper, it is assumed that all these securities are bundled in a portfolio of domestic and foreign assets and that this portfolio spans all the states of the world. This is made in order to have an easy mapping from the model into the data set provided by Gourinchas and Rey (2007a) (more on this below). The assumption about the single agent excludes the possibility of domestically-run Ponzi

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\(^8\)The law of one price is implicitly assumed as well.  
\(^9\)The whole fiscal policy is basically treated as exogenous in this simple setup. It would be fairly easy to extend the model in order to consider a welfare-maximizer, benevolent government by including \( G_t \) in the utility function of the household. If the functional form of the utility is maintained separable, the implications for the empirical definition of the discount factor would be the same.  
\(^10\)\( P^H_t \) is the price of the domestic good relative to the composite good, \( \frac{P^{H}}{Y} \). By national accounting definition, \( P^H_t Y_t \) corresponds to the real value of the factors’ remuneration, which is simply the households’ wage bill in a model with no capital. The term \( C_t + G_t \) is the demand absorption of GDP. It would also include investment in a model with capital.
schemes, each country tries to run Ponzi games only against the other country at most. Although this is
a simplification because, for example, a national government can always default on debt held by its own
citizens, this assumption allows to focus the attention only on the sustainability of the overall external debt
of a country, which is the goal of this paper.

The constraint in (8) can be written in terms of foreign assets and liabilities as

\[ B_t = L_t - A_t = (1 + r^L_t) L_{t-1} - (1 + r^A_t) A_{t-1} - D_t \]  

where \( B_t \) represents the net foreign liabilities position of the domestic country in terms of the domestic
composite good, given by the difference of foreign liabilities \( L_t \) and foreign assets \( A_t \), and \( D_t \) is the trade
balance. This equation closely resembles the standard definition of the current account, but, as discussed
above, it also takes into account the different returns on assets and liabilities and the valuation effects. In
fact, \( r^A_t \) includes the valuation effects due to the exchange rate movements, while both \( r^A_t \) and \( r^L_t \) includes
also the standard capital gains due to changes in the assets prices.

Under these assumptions, the remaining features of a model are of secondary importance for the char-
acterization of the economy-wise budget constraint and the transversality condition. Different assumptions
about the utility function of the consumer, the presence of labor and capital, or the role of the government
would bring to a budget constraint for the domestic economy of the same form as in (9). \(^{11}\)

From asset pricing under complete markets and the Euler equations of the representative household/agent,
the unique stochastic discount factor that determines the prices of the assets for given structure of payoffs
coincides with the intertemporal ratio of marginal utilities

\[ \mu_{t+1} = \beta \frac{U'(C_{t+1})}{U'(C_t)} = \beta \frac{C^\pi_{t+1}}{C^\pi_t} \]

\(^{11}\)In order to get to (9), let define the portfolio holdings as \( A_t = S_t P^A_t \tilde{A}_t \) and \( L_t = P^L_t \tilde{L}_t \) respectively; where \( P^A_t \) and \( P^L_t \) are the real prices of the assets in terms of local composite goods, \( \varphi^A_t \) and \( \varphi^L_t \) are the payoffs of the assets, and \( S_t \) is the real exchange rate between the two countries. These definitions imply that \( \tilde{A}_t \) and \( \tilde{L}_t \) are the numerical amounts of assets and liabilities held by the domestic agent. Equation (8) can be stated now as

\[ C_t + G_t + S_t P^A_t \tilde{A}_t - P^L_t \tilde{L}_t = \left( S_t P^A_t + S_t \varphi^A_t \right) \tilde{A}_{t-1} - \left( P^L_t + \varphi^L_t \right) \tilde{L}_{t-1} + P^H_t Y_t \]  

which, after a few manipulations, returns (9). In particular, the following definitions of the assets returns \( r^A_t \) and \( r^L_t \) are used

\[ (1 + r^A_t) = \frac{S_t}{S_{t-1}} \left( \frac{P^A_t + \varphi^A_t}{P^A_{t-1}} \right) = \frac{P^A_t}{P^A_{t-1}} + \frac{\varphi^A_t}{P^A_{t-1}} \]

\[ (1 + r^L_t) = \frac{P^L_t}{P^L_{t-1}} + \frac{\varphi^L_t}{P^L_{t-1}} \]

Notice also that, by construction, \( P^A_t = S_t P_t^A \) and \( \varphi^A_t = S_t \varphi_t^A \).
Equation (9) can be expressed then as

\[ B_t = E_t \left[ \mu_{t,t+1} \left( 1 + r^L_{t+1} \right) L_t \right] - E_t \left[ \mu_{t,t+1} \left( 1 + r^A_{t+1} \right) A_t \right] \]

\[ = \left( 1 + r^L_t \right) L_{t-1} - \left( 1 + r^A_t \right) A_{t-1} - D_t \]  

(10)

where use has been made of the portfolio condition of the representative agent in (11)\textsuperscript{12}

\[ E_t \left[ \mu_{t,t+1} \left( 1 + r^i_{t+1} \right) \right] = 1 \quad i = A, L \]  

(11)

Equation (10) shows that the correct recursive structure of the budget constraint is defined on the net positions inclusive of the returns payments, \((1 + r^L_t) L_{t-1} - (1 + r^A_t) A_{t-1}\), rather than simply \(L_t - A_t\). Recursively substituting this term in (10), one obtains the intertemporal budget constraint

\[(1 + r^L_t) L_{t-1} - (1 + r^A_t) A_{t-1} = E_t \sum_{i=0}^{\infty} \mu_{t,t+i} D_{t+i} \]  

(12)

where \(\mu_{t,t+i} = \prod_{j=1}^{i} \mu_{t+j-1,t+j} \) and \(\mu_{t,t} = 1\).

Equation (12) defines the sustainability of the debt position. In agreement with the intertemporal approach of the current account, debt must be financed by positive-in-expectation future discounted trade surpluses. Technically, positive \(D_{t+i}\) are not even strictly necessary if the the covariance between the discount factor and the trade surplus is negative and large enough. In general, however, this condition justifies the statement that some positive future trade surpluses are required to repay an initial debt.

For equation (12) to hold, it is essential that the following transversality condition holds too:

\[ \lim_{T \to \infty} E_t \left[ \mu_{t,t+T} \left( 1 + r^L_{t+T} \right) L_{t+T-1} - \mu_{t,t+T} \left( 1 + r^A_{t+T} \right) A_{t+T-1} \right] = 0 \]  

(13)

The limit of the expected discounted net foreign debt, inclusive of interest payments, must be equal to zero. The condition in (13) closely resembles the standard transversality condition of a perfect foresight model. However, the use of an optimal stochastic discount factor shifts the attention from the simple value of the net debt to the fully specified future payoffs of assets and liabilities. If perfect foresight had been assumed and if the returns on the two assets had been the same, then (13) would coincide again with the basic transversality

\textsuperscript{12}Equivalently, equation (10) and (11) can be restated using the notation of equation (N1) as

\[ E_t \left[ \mu_{t,t+1} \left( P^L_{t+1} + \phi^L_{t+1} \right) \right] \tilde{L}_t - E_t \left[ \mu_{t,t+1} \left( P^A_{t+1} + \phi^A_{t+1} \right) \right] \tilde{A}_t = \left( P^L_t + \phi^L_t \right) \tilde{L}_{t-1} - \left( P^A_t + \phi^A_t \right) \tilde{A}_{t-1} - D_t \]

\[ E_t \left[ \mu_{t,t+1} \left( P^L_{t+1} + \phi^L_{t+1} \right) \right] = P^L_t \quad i = A, L \]
The condition in (13) is an equilibrium implication of the optimal behavior of each optimizing agent, who avoids the other agent’s Ponzi games, but, at the same time, tries to exploit any opportunity to run a Ponzi scheme against the other country.

3 A Description of the Data

In the empirical analysis of the next section I use quarterly data on trade from the National Income and Product Accounts (NIPA) tables and the data from Gourinchas and Rey (2007a) for the financial series.

The NIPA tables published online by the Bureau of Economic Analysis (BEA) provide the nominal series of imports, exports, international income payments and receipts, GDP, and consumption. This is essentially all we need to construct the nominal trade balance, the discount factor, and the ratio to GDP series that are used as counterparts of the real variables in Section 2. The consumer price index series from the online database of the Federal Reserve Bank of St. Louis, the Federal Reserve Economic Data (FRED), is used to construct the real variables. Gourinchas and Rey (2007a) provide the series of the U.S. nominal foreign assets and liabilities holdings; they also provide the valuation rates and the total implicit nominal rates of return for the two categories of international assets.\(^\text{14}\)

The common sample covered by both sources dates back to the beginning of the 50s up to 2004, precisely from 1952:3 to 2004:1. The samples of the VAR regressions are then limited by the real exchange rate series, which can be obtained only since the 70s. I use the series constructed by Francesco Bianchi and Andrea Civelli (09), which is very similar to the major currencies real exchange rate series reported by FRED, but has the advantage of starting from 1970:1. All the series are annualized, seasonally adjusted whenever necessary, and transformed into real terms as described in Section 4.

The timing of the observation of the data needs a further clarification, in particular because this is important for the following empirical applications. Gourinchas and Rey provide the end of period nominal values of the portfolio allocations, computing the flows of assets and valuation changes over the period. In the notation of equation (14), which is drawn from their paper, \(b_{t+1}\) refers to the end of period \(t + 1\) value of net assets, while \(\Delta f_{t+1}\) are the flows accrued during period \(t + 1\) (that is from the end of period \(t\), when

\[ \mu_{x,t+1} = (1 + r_{t+1})^{-1} \text{ for any time } t. \]

The last term of the multi-period discount factor, \((1 + r_{t+T})^{-1}\), would cancel out with the \((1 + r_{t+T})\) that multiplies the \((t + T - 1)\) assets position and the transversality condition becomes the familiar

\[ \lim_{T \to \infty} \mu_{t,t+T-1}B_{t+T-1} = 0 \]

\(^{13}\)In this case \(\mu_{x,t+1} = (1 + r_{t+1})^{-1}\) for any time \(t\). The last term of the multi-period discount factor, \((1 + r_{t+T})^{-1}\), would cancel out with the \((1 + r_{t+T})\) that multiplies the \((t + T - 1)\) assets position and the transversality condition becomes the familiar \(\lim_{T \to \infty} \mu_{t,t+T-1}B_{t+T-1} = 0\).

\(^{14}\)The empirical analysis of this paper has been made possible only by the availability of Gourinchas and Rey’s data set, which is published on the authors’ webpages. The reader is invited to make reference to their paper for a detailed description of these data.
has been observed, to the end of period $t + 1$).

$$b_{t+1} = b_t + \Delta f_{t+1} + x_{t+1}b_t$$  \hspace{1cm} (14)$$

The same timing of $\Delta f_{t+1}$ must be assumed for the NIPA data of trade balance and income payments and receipts, since $\Delta f_{t+1}$ conceptually corresponds to the current account. Finally, the real exchange rate $S_t$ and the valuation rate $x_t$ are observed at the end of the quarter, as well as the level of the consumer price index $P_t$, the prices of assets and liabilities $P_t^A$ and $P_t^L$, and the growth rate of GDP used in the next section to construct the real terms version of the budget constraint.

The limitation in the sample suitable for the VAR regressions due to the real exchange rate series is relatively unimportant. In some sense, it helps to mitigate possible structural breaks in data that may have followed the increase in openness of the world economy after the collapse of the Bretton Woods system. However, the results I obtain excluding the real exchange rate from the basic specification of the VAR do not seem particularly affected by the different sizes of the samples.

4 An Empirical Assessment of the U.S. Net Foreign Debt

4.1 Empirical Evaluation of the transversality condition for the period 1989-2004

For about four decades, since the beginning of the sample covered by the data, the U.S. have been holding a positive and quite stable net assets to GDP ratio. By the end of the 80s, not only did they switch from being a net lender to being a net borrower, but also the net debt to GDP ratio started increasing at a faster pace. In this section, I empirically show that the dynamics of the net foreign debt of those years violates the transversality condition on the economy-wise budget constraint derived in Section 2.1. In the next sections, I assess the role of Gourinchas and Rey’s (2007b) valuation effects in the sustainability process of the U.S. international debt and I conclude that it would be a mistake to rely on the valuation channel in order to achieve sustainability.

In order to show this, I evaluate the conditional expectations of the limit on the left hand side of condition (13), given the implication for the discount factor of the theoretical model. For convenience of notation, let $BB_t$ indicate the debt inclusive of interest payments

$$BB_t = (1 + r_t^L) L_{t-1} - (1 + r_t^A) A_{t-1}$$
We can write the limit in a more compact way that can be conveniently exploited to make an empirical evaluation of the transversality condition. The term in brackets in (13) is equivalent to

\[ \beta^T \frac{U_{c,t+T}}{U_{c,t}} BB_{t+T} = BB_t \left[ \beta \frac{U_{c,t+1}}{U_{c,t}} ... \beta \frac{U_{c,t+T}}{U_{c,t+T-1}} \right] \left[ \frac{BB_{t+1}}{BB_t} ... \frac{BB_{t+T}}{BB_{t+T-1}} \right] = \]

\[ = BB_t \left[ UB_{t+1} UB_{t+2} ... UB_{t+T} \right] \]

where \( UB_{t+1} = \beta \frac{U_{c,t+1}}{U_{c,t}} BB_{t+1} \). The variable \( UB_t \) can be easily constructed from the data and used to evaluate the conditional expectations

\[ BB_t \lim_{T \to \infty} \mathbb{E}_t [UB_{t+1} UB_{t+2} ... UB_{t+T}] \]

Intuitively, for given \( BB_t \), which drops out of the conditional expectations operator, the limit converges to zero if the realizations of \( UB_t \) are on average smaller than 1. More formally, the law of iterated expectations can be progressively applied to the terms in brackets in (15), beginning from the last period and back to the first, in order to obtain an expression of nested one period ahead conditional expectations of each of them. For instance, the procedure for just the last two periods would return

\[ E_t \{ UB_{t+1} UB_{t+2} ... E_{t+T-2} [UB_{t+T-1} E_{t+T-1} (UB_{t+T})] \} \]

and so on until period \( t+1 \). If the series \( UB_t \) is not auto-correlated, each conditional expectation is equivalent to the unconditional one \( E_{t-1} (UB_t) = E (UB_t) \) and the overall conditional expectation in (16) is simply equivalent to \( E (UB_t)^T \). This term converges to zero only if \( E (UB_t) < 1 \) and an estimate of \( E (UB_t) \) can be obtained from its sample mean.

I construct the real series for \( C_t, r_t^L, r_t^A, L_t, A_{t-1} \) deflating the nominal series by the consumer price index. Figure 1 shows the auto-correlogram of \( UB_t \) for the sample 1989:3-2004:1. Assuming that \( UB_t \) is not auto-correlated seems a legitimate assumption since the auto-correlogram is basically flat, except for an isolated, marginally significant spike at the fifth lag. Under a standard calibration for the intertemporal discount factor and the coefficient of relative risk aversion in the utility function (\( \beta = 0.99 \) and \( \sigma = 2 \)), the sample mean of \( UB_t \) is 1.063 (with standard deviation 0.038), which implies that the observed net debt series has been moving along a non-sustainable path.\(^{15}\)

This does not allow one to conclude that the U.S. net debt position is not sustainable yet; however, it

\(^{15}\)Lower values of \( \beta \) and higher values of \( \sigma \) would decrease the mean of \( UB_t \) by lowering the average discount factor. However, values of \( \sigma \) around or larger than 10 are necessary to make it fall below 1. Although there is not a general consensus about the estimates of the coefficient of risk aversion, 10 would be normally considered a quite high, albeit plausible, value of \( \sigma \).
Figure 1: Auto-Correlogram of $UB_t$

Notes: Sample 1989:3-2004:1. The "Q-Stat" column refers to the Ljung-Box test for the null hypothesis that there is not auto-correlation up to the $K^{th}$ lag; the p-value of the statistic is reported in the column "Prob". The column labeled "AC" ("PAC") is the auto-correlation (partial auto-correlation) of the series.

suggests at least two other important observations. First, a strong correction of the accumulation process of the international liabilities would definitely be required to re-equilibrate this situation. Second, since the debt series already includes the valuation effects, we can question their actual contribution to sustainability. They can obviously help stabilizing the debt series, but a further investigation of their full role is necessary.

The worrying acceleration of the net debt accumulation is evident if we compare the post 1989 mean to the sample mean before 1988, when the U.S. was holding positive net assets. Formally, the sustainability was not an issue because there was not debt, but the U.S. was already decumulating assets. Hence, this exercise must be taken as an empirical, rather than a formally correct, comparison. We can use the same logic to compute the rate of shrinking of assets, corresponding to the ratio $BB_{t+1}/BB_t$ of the post 1989 period, and find the mean of $UB_t$ for the first decades of the sample. This mean is 0.981 (s.d. 0.007) and it reflects the fact that, even though the debt accumulation had already began, the net assets process was not raising any particular concerns yet.\textsuperscript{16}

Even though it is only a sufficient condition for sustainability, it is interesting to compute an estimate of $\alpha$ for the type of test in equation (5). I report the simple OLS estimates (including a constant too) for

\textsuperscript{16}Noted also that $BB_{t+1}/BB_t$ is correctly defined only if $BB_t \neq 0$ and it does not change of sign. Therefore, I drop the observation between 1988:1 and 1989:2, when the sign switches back and forth from assets to liabilities for a few times in order to avoid explosive ratios simply due to small denominators.
the largest available sample and the same sub-sample considered in this section. In the regression, I use the ratios to GDP of the deficit and debt variables, \( d \) and \( b \) respectively, as defined in the next section. In both cases, the estimates are strongly significant and have the wrong sign, which reinforces the doubts about the sustainability of \( b \). For the full sample \( \hat{\alpha} = -0.11 \) (with a small standard error of 0.006), while for the latest sample I obtain \( \hat{\alpha} = -0.14 \) (.008). This piece of evidence is not completely conclusive, but it clearly points in the same direction as the other results in this section.

### 4.2 The Debt to GDP Ratio

For the empirical analysis in the rest of the paper, I consider the debt to GDP ratio, instead of the real debt obtained by dividing the nominal debt by the price level. This is done because it is common practice in literature to address the questions about debt sustainability looking at this ratio since the level of income is considered a correct reference level for debt. The same standardization is used in Bohn (1998), for example, and in most of the literature I reviewed in Section 1; a similar correction is adopted by Gourinchas and Rey (2007b) who take the ratio of debt to total wealth.

As in Section 1, let the lower cases indicate the ratio of nominal variables to GDP. The corresponding budget constraint is

\[
b_t = (1 + r^d_t) h_{t-1} - (1 + r^a_t) a_{t-1} - d_t
\]

where the return rates \( r^d_t \) and \( r^a_t \) are real returns that incorporate the growth rate of GDP. It is worth stressing again that the asset returns depend on the depreciation rate of the real exchange rate too, denoted \( \gamma \) henceforth.\footnote{It is easy to link these return rates to the real returns in Section 2. Denoting \( g_t \) the real growth rate of GDP, we can express \( r^d_t \) and \( r^a_t \) as

\[
(1 + r^d_t) = \frac{(1 + r^A_t)}{(1 + g_t)} = \frac{(1 + \gamma_t) (1 + r^A_t)}{(1 + g_t)}
\]

\[
(1 + r^a_t) = \frac{(1 + r^A_t)}{(1 + g_t)}
\]

where it should be clear from the notation adopted so far that \( r^A_t \) is the real return rate of the foreign assets in terms of foreign goods.}

It must be noticed that considering the ratios to GDP of the variables in the budget constraint does not invalidate the relevant transversality condition in (15) because restating the problem in GDP terms would determine also a change in the definition of the discount factor. It can be shown that the transversality conditions in the two cases are equivalent up to a multiplicative scaling factor, therefore the analysis of section 4.1 remains valid.\footnote{This scaling factor is simply the inverse of the real output \( Y_t \), which is the ratio between \( BB_t \) and \( bb_t \).}

Before turning to the VAR analysis, I check for the stationarity of the series included in the budget
constraint, which appear in the transversality condition and the VAR too. The augmented Dickey-Fuller test for unit roots is used; Table 1 reports the results of the tests.\textsuperscript{19}

The top panel of Table 1 shows the results for assets and liabilities, trade balance and real exchange rate; the bottom panel for the debt inclusive of interest payments, $bb_t = (1 + r^d_t) l_{t-1} - (1 + r^p_t) a_{t-1}$, and without interest payments, $b_t$, under alternative specifications of the test. All the series, with the exception of the real exchange rate whose statistic is marginally significant, seem to be non-stationary and the unit root tests are not rejected with large margins. The same conclusions are confirmed also for almost all the other specifications of the Dickey-Fuller tests too.

The tests on the debt series aim to provide some evidence on its stationarity, rather than a proof of the existence of the unit root per se. As seen in Section 1, there is an important theoretical consideration that should prevent us from assuming that the debt series is $I(1)$ because a stationary, or even integrated, debt would make the transversality condition hold by construction. Here, the crucial point is to understand whether the debt is growing at a faster rate than that implied by the relevant discount factor, which seems to be the case since 1989, as just seen above. However, since in the VAR literature it is a common first step to check for the integration of the time series included in the regression and given that the random walk is

\textsuperscript{19}Hamilton (1994) describes the procedure to follow in order to choose the specification of the test equation and provides the correct tabulation of the values of the $t$-test statistic in each case. The test is based on a regression which, in its most general form, has the following specification:

$$y_t = \lambda_1 \Delta y_{t-1} + \lambda_2 \Delta y_{t-2} + \ldots + \lambda_{p-1} \Delta y_{t-p+1} + \alpha + \rho y_{t-1} + \delta t + \varepsilon_t$$

I select the lags $p$ according to the following procedure: From the auto-correlogram of $y_t$, one can single out the highest value of $p$ such that the $(p-1)\text{th}$ term in the test equation is presumably not significant, this $p$ should correspond to the first lag at which the auto-correlogram fades out. Given this, the joint significance of the $(p-1)\text{th}$ and the $(p-2)\text{th}$ terms in the regression is tested: If the F-test does not reject the significance of the two terms, the procedure stops here and the $\Delta y_{t-p+1}$ is included in the test regression. If, on the other hand, the test rejects it, $\Delta y_{t-p+1}$ is discarded and the same steps are repeated for the next lag.
the baseline case of non-stationarity, it is worthwhile to run these tests. Finally, estimating the VAR with Bayesian techniques overcomes the issues related to the presence of unit roots in the variables of the model, at least for the correct computation of the impulse response functions and their bands.\textsuperscript{20}

5 Predictability of Excess Returns and Trade Balance Dominance

Gourinchas and Rey (2007b) show that a measure of trade and debt imbalances has a significant predictive power on the future excess returns of the American international portfolio. The intertemporal budget constraint in (12), however, has the theoretical implication that the current net foreign liabilities should predict future trade surpluses, but not the excess returns. This is an apparent contradiction that I discuss in this section in light of the framework of this paper.

In a stochastic environment, the returns rates are endogenously tied to the stochastic discount factor by the optimal portfolio choices of the representative agent. From a general equilibrium perspective, the returns are embedded in the recursiveness of the forward solution of the net asset position derived from the economy-wise budget constraint, as shown in Section 2.1, and the intertemporal budget constraint should not bear any information about future excess returns.

Gourinchas and Rey’s results, in particular the systematic contribution in the short to medium-run of the excess returns to the rebalancing mechanism of the debt accumulation process, can be given an intriguing interpretation in terms of the traditional Ricardian theory of fiscal policy. If temporarily loose trade surpluses, which are for the foreign debt of a country the same as the budget surpluses for the government debt, are systematically compensated by favorable excess returns, these trade balances show, at least in part, some non-Ricardian feature. This sort of trade balance dominance is achieved through a self-adjusting mechanism in which the depreciation of the real exchange rate triggers the positive excess returns while preparing the way to the long-run positive surpluses.

Their results, however, are found in the context of a linearized version of a deterministic model, after imposing the transversality condition. This transformation of the data has an important effect on the interaction among the variables of the system. It drops second and higher order effects related to the riskiness of the assets that do have non-negligible implications in general equilibrium for the portfolio allocation and the overall sustainability of the debt position.

A first simple intuition can be provided by a deterministic model in levels and, hence, by construction with no risk. In such a model, the discount factor is constant and coincides with the risk-free real rate in

\textsuperscript{20}I return to this last point in the next section. This paper helps in stressing the point that the cointegration analysis is probably not the most convenient approach to model the relevant long-run relationships among the variables in this VAR when the goal is to discuss the sustainability of debt. For a very comprehensive cointegration study of the adjustment dynamics of the US current account, see for example Mark J. Holmes and Theodore Panagiotidis (2009).
the economy. For example, with perfect foresight, the discounting rate relevant for the optimizing agent can be assumed to be the interest rate paid on his own liabilities. Solving (17) forward for \( b_{t-1} \), one obtains

\[
b_{t-1} = \sum_{i=0}^{\infty} \Gamma^L_{t+t+i} a_{t+i} + \sum_{i=0}^{\infty} \Gamma^L_{t+t+i} r^x_{t+i} a_{t+i-1}
\]

where \( r^x_t = (r^A_t - r^L_t) \) is the excess return on the international portfolio, the discount factor between time \( t \) and \( t+i \) is \( \Gamma^L_{t,t+i} = \prod_{k=t}^{t+i} (1 + r^L_k)^{-1} \), and the transversality condition \( \lim_{T \to \infty} \Gamma^L_{t, t+T} b_{t+T} = 0 \) has been imposed. In this case, the current net debt is the discounted sum of future trade surpluses and positive excess returns multiplied by the gross asset position.

The linearization of the budget constraint, even in a stochastic model, has the same kind of effect on how the evaluation of risk is factored into the dynamics of the model. It is easy to see that (10) can be log-linearized around a steady state in which \( A_{ss} \neq L_{ss} \) and \( D_{ss} \neq 0 \), but with the same steady state returns rates, to obtain

\[
\theta \tilde{L}_t - (\theta - 1) \tilde{A}_t = \frac{1}{\beta} \left[ \theta \tilde{L}_{t-1} - (\theta - 1) \tilde{A}_{t-1} \right] - \frac{1}{\beta} \left[ (\theta - 1) \tilde{R}^A_t - \theta \tilde{R}^L_t \right] - \delta \tilde{D}_t
\]

where \( \tilde{R}_t^d = \left( 1 + r^d_t \right) \), \( \theta \) and \( \delta \) are the steady state ratios \( \theta = \frac{B_{ss}}{A_{ss}} \) and \( \delta = \frac{D_{ss}}{B_{ss}} \), and the overscore hat indicates the log-deviations from steady state. Defining the debt deviations as \( \tilde{B}_t = \theta \tilde{L}_t - (\theta - 1) \tilde{A}_t \) and the excess returns deviations as \( \tilde{R}^x_t = (\theta - 1) \tilde{R}^A_t - \theta \tilde{R}^L_t \), the linearized constraint can be solved forward for \( \tilde{B}_{t-1} \)

\[
\tilde{B}_{t-1} = \sum_{i=0}^{\infty} \beta^i \tilde{R}^x_{t+i} + \delta \sum_{i=0}^{\infty} \beta^{i+1} \tilde{D}_{t+i}
\]

The inverse of the steady state return rate, \( \beta \), is now the discounting factor and again the current net debt is the sum of future excess returns and trade surpluses as in Gourinchas and Rey.\(^{21}\)

### 5.1 Evidence from VAR

In this section, I check whether the implications of the two versions of the model are empirically supported by the data. This task is accomplished by estimating a Bayesian VAR model and by computing the impulse response functions of the excess returns to shocks to the debt position and, in a smaller extent, to trade surplus.

In this section, I check whether the implications of the two versions of the model are empirically supported by the data. This task is accomplished by estimating a Bayesian VAR model and by computing the impulse response functions of the excess returns to shocks to the debt position and, in a smaller extent, to trade surplus.

\(^{21}\)Notice also that the condition \( \lim_{T \to \infty} \beta^i \tilde{R}_{t+T-1} = 0 \) has been applied. This is not exactly a transversality condition; if the linearization is around a stationary steady state, for example, this condition would be satisfied by construction.
Figure 2: Response functions to a \( bb \) unit shock. VAR in levels

Notes: Bands Intervals correspond to the 14/86\text{th} and 5/95\text{th} percentiles. Years from the impulse on the \( x \)-axis.
Figure 3: Response functions to a $d$ unit shock. VAR in levels

Notes: Bands Intervals correspond to the $14/86th$ and $5/95th$ percentiles. Years from the impulse on the $x$–axis.
response functions of the excess returns to shocks to the debt position and, in a smaller extent, to trade surplus.

As we move from the model in levels toward the counterpart of the linearized model, we observe a radical change in the shape of these response functions. This framework allows to conclude that, even though they can actually play an active role in the dynamics of the international portfolio position at higher frequencies, the valuation effects are not systematically involved in the sustainability of the U.S. net debt position.

Furthermore, a separate analysis of the individual responses of the domestic and foreign returns shows that the valuation effects are only in part related to the movements of the exchange rate. Other macroeconomic factors of risk, as for example productivity or demand shocks which are not explicitly included in this analysis, but typically used in theoretical models as exogenous innovations, seem to combine together in the determination of the capital gains. From the perspective of sustainability, the role of the depreciation of the exchange rate, which is considered the trigger of the favorable excess returns found in Gourinchas and Rey, must be limited mostly to the real effects it has on the trade balance.

A caveat is necessary here. These conclusions do not mean that real exchange rate depreciations and positive excess returns do not determine the net debt position at all; the debt at each point in time is the accumulation of the previous assets flows and valuation gains and losses. However, they do suggest that there is no particular content, in terms of sustainability, in those movements of the exchange rate. In practice, this idea boils down to the difference between sustainability and stability of the debt position. The valuation effects can help to stabilize the debt, for example at business cycle frequencies, but it turns out that they do not systematically support its sustainability. In terms of the budget constraint equation, which, after all, is a differences equation, this is equivalent to considering a forward-looking versus a backward-looking solution of the debt.

5.1.1 Model in Levels

I include four variables in the basic specification of the VAR. These variables are the net foreign debt position inclusive of interest payments $bb_t$, the trade surplus $d_t$, the excess returns $r^x_t = r^a_t - r^l_t$, and the depreciation rate of the real exchange rate $\gamma_t$. In the following set of results, I estimate the VAR with two lags using a Minnesota-type prior over the sample 1970:1-2004:1. The parameter of the tightness for the own lag of a variable is set to 0.2, the relative tightness parameter for the other variables’ lags is set to $10^3$. The lags have an harmonic decay with parameter 2, and the tightness parameter of the exogenous variables (only the constant here) is set to $10^3$. The prior mean of the first lag is 1 for $bb$ and $d$, 0 for $r^x$ and $\gamma$. The structural shocks for the impulse response functions are identified with a recursive Cholesky scheme with baseline ordering $(d, bb, \gamma, r^x)$. I leave a detailed discussion of the choice of the prior coefficients and the
Figure 4: Variance Decomposition for the VAR in levels

Notes: Horizon of the forecast in years on the $x$–axis. Ordering of the variables is \([d \ b b \ \gamma \ r^x]\).
Figure 5: Separating $r^a$ and $r^l$. VAR in levels

Notes: Separate Responses of $r^a$ and $r^l$ to the $bb$ and $d$ shocks (upper and lower panel). VAR in Levels with $r^a$ and $r^l$ replacing $r^x$. Years from the impulse on the $x$–axis.
Figure 6: Response functions to a \( bb \) unit shock. Gourinchas and Rey’s de-trending

*Notes:* Variables de-trended as in Gourinchas and Rey (2007b). Bands Intervals correspond to the 14/86th and 5/95th percentiles. Years from the impulse on the \( x- \) axis.
Figure 7: Response functions to a $d$ unit shock. Gourinchas and Rey’s de-trending

Notes: Variables de-trended as in Gourinchas and Rey (2007b). Bands Intervals correspond to the $14/8th$ and $5/95th$ percentiles. Years from the impulse on the $x-$axis.
sensitivity analysis of the results for the next section and the Appendix.

Figure 2 shows the response functions of the four variables to a unity impulse to $bb$ for the model in levels. The two pairs of dotted lines correspond to the $14/86$th and $5/95$th percentile intervals of the posterior distribution of the response functions computed by Monte Carlo integration. At impact, the real exchange rate appreciates leading to a large negative response of the excess return; after one quarter, however, $r^x$ promptly reverts toward zero and remains not significant since then on. Two years after the shock the median response of $\gamma$ and $r^x$ becomes slightly positive, but their bands are quite narrow and wrap the zero line. The response of the trade balance, on the other hand, is initially positive for about four to eight quarters. Then, it turns negative in a very persistent way and only the upper tail of the $5/95$th band remains in the positive region.

These two responses are potentially consistent with an ultimate sustainability of the debt trajectory if the responses of $d$ are strong enough and provide evidence against the possibility of having trade surplus dominance in the Ricardian sense. The initial response of $d$ and the lack of predictable valuation gains are perfectly in line with the theoretical predictions based on the intertemporal budget constraint (12) under the assumption of Ricardian trade balances, but in the medium long-run the response of $d$ is very weak. A small positive probability of observing future increases of the trade surplus is still there, but the prolonged median deficits leave room to accelerations of the debt accumulation along an unsustainable path, of the type documented in Section 4.1.

Figure 3 reports the response functions to a unity impulse to $d$. The response of $\gamma$ and $r^x$ are similar to those for the $bb$ shock, but slightly more pronounced.\textsuperscript{22} This may be interpreted as suggesting some sort of predictability of the excess returns, but, actually, this is not a very important effect. If we combine this piece of evidence with the variance decomposition associated with these impulse response functions in Figure 4, it is clear that the share of variance of $r^x$ explained by the $d$ shocks is absolutely irrelevant. We can also observe from Figure 4 that about 60 percent of the variance of $r^x$ is attributed to the $bb$ shocks and that, as the forecast horizon increases the reciprocal importance of $d$ and $bb$ increases too.

The valuation channel is based on the dependence of the realizations of $r^a$ on the exchange rate. In Figure 5, I consider the separate response functions of $r^a$ and $r^l$ to the $bb$ and $d$ shocks in order to evaluate the relative contribution of the two to the excess return. A VAR analogous to the previous one, but with the two returns, is estimated. Three observations about this picture are interesting. First, the two rates show very close comovements, in particular at shorter horizons. Second, the responses of $r^a$ follow, as expected, the movements of the real exchange rate quite well, even though they are more amplified. Third, the responses of $r^l$ are not small at all and the contribution of $r^l$ to $r^x$, in particular in the short to medium-run, seems

\textsuperscript{22}I will consider the response of $bb$ in the next section, when I talk about the Ricardian regimes of the real exchange rate.
to be quite relevant. For example, in the upper panel of Figure 5, we can see that the response of $r^j$ to a $bb$ shock is persistently negative and this contributes to generate a small positive (although not significant) excess return in Figure 2. This component cannot be directly attributed to the exchange rate valuation channel per se.

This kind of evidence shows that the transmission of the shocks to the return rates of international assets is probably very articulated. The valuation channel is an important element in the determination of the composition of the excess returns because the exchange rate depreciations have a direct impact on $r^a$, but then other significant contributions to transmission must be taken into account. My setup does not model these other channels explicitly, but many suggestions in this respect can be easily found in the theoretical open economy RBC literature.

![Variance Decomposition of d](image1)

![Variance Decomposition of bb](image2)

![Variance Decomposition of γ](image3)

![Variance Decomposition of $r^x$](image4)

Figure 8: Variance Decomposition for the Gourinchas and Rey’s de-trending

Notes: Variables de-trended as in Gourinchas and Rey (2007b). Horizon of the forecast in years on the $x$–axis. Ordering of the variables is $\left[ d \ b b \ \gamma \ r^x \right]$.

5.1.2 Linearized Model

I turn next to the empirical counterpart of the linearized version of the model. Instead of considering all the variables in levels, the two non-stationary variables of the VAR, $d$ and $bb$, are de-trended and only their
Figure 9: Separating $r^a$ and $r^l$. Gourinchas and Rey’ de-trending

*Notes:* Separate Responses of $r^a$ and $r^l$ to the $bb$ and $d$ shocks (upper and lower panel). Variables de-trended as in Gourinchas and Rey (2007b) with $r^a$ and $r^l$ replacing $r^x$. Years from the impulse on the $x$–axis.
stationary components around the trend are included in the estimated VAR. This is a common strategy used, for example, in the RBC literature, but I adopt it, in particular, because this is the same approach followed by Gourinchas and Rey (2007b). The excess returns and $\gamma$, on the other hand, are already stationary and do not need to be de-trended.

It is not obvious that de-trending is the most appropriate approach to deal with possible long-run relationships among the variables of the VAR. The main alternative is typically to use a Vector Error Correction Model (VECM), which exploits cointegration relations between integrated series. Although also admissible in this context, there is an important theoretical issue that prevents the simple use of the cointegration approach here. The net debt series is clearly non-stationary, but imposing a unit root to it would be equivalent to imposing the transversality condition to the intertemporal budget constraint, which is actually the ultimate goal of my analysis.\textsuperscript{23}

There are numerous alternative ways to de-trend a time series. Gourinchas and Rey suggest to compute the trends using an Hodrick-Prescott filter with a very high smoothing parameter, which makes the trend almost linear, in order to get rid only of the very slow-moving components of the series. I present the results corresponding to their choice of the parameter in Figure 6-9.\textsuperscript{24}

Figure 6 shows that the responses at impact of $\gamma$ and $r^x$ to a $bb$ shock remain the same as in the baseline case, but now this shock is followed by prolonged and significant excess returns. These response functions are in line with the predictive regressions in Gourinchas and Rey (2007b), but, at the same time, confirm the theoretical implications discussed at the beginning of this section. A larger portion of the distribution of the trade balance response is in the positive region too, but the contribution of $r^x$ in keeping $bb$ under control seems to be essential. In fact, estimating the same VAR with a tighter prior on the parameter controlling the relative tightness of the other variables’ lags would make the positive excess return disappear and the debt non-stationary again. This point is fully discussed in the next section and the Appendix.

I also try a smaller and more conventional smoothing parameter, which isolates the business cycle frequencies of the series, as typically done in the RBC models. The response functions in this case fade out faster than in the Gourinchas and Rey’s scenario, but the overall, core message does not change.\textsuperscript{25}

The relation between the real exchange rate depreciation and $r^x$ deserves some more attention. As I said, the valuation effects are typically attributed to movements of the exchange rate and the returns on the asset position, and this is the interpretation given by Gourinchas and Rey too. The responses in Figure

\textsuperscript{23}It is possible to find a stationary combination of the non-stationary variables of a VAR model also when these variables are not $I(1)$. The VECM representation, however, should not be valid in this case. I do not explicitly investigate this possibility in this paper.
\textsuperscript{24}The smoothing parameter chosen by Gourinchas and Rey sets a gain of .7 at the frequency of 200 quarters. This corresponds to the value of 2400654. On the other hand, the standard value used in the RBC literature for quarterly series would be 1600.
\textsuperscript{25}The plots for this last case are not reported here, but are available from the author.
show again a different situation. For example, even if the exchange rate depreciates in a quite evident way in response to a \( bb \) shock in the Gourinchas and Rey’s setup, \( r^a \) in the upper panel of Figure 9 does not increase much. It is the return rate on the liabilities, \( r^l \), that, after an initial increase, turns negative and drives the excess return we observe in Figure 6. Even when \( r^a \) shows more significant and pronounced positive responses, as in the case of the business cycle de-trending, \( r^l \) is always playing a primary role in the determination of the excess return. These last observations reinforce the conclusions presented at the end of Section 5.1.1 for the VAR in levels.

5.2 Sensitivity Analysis and Choice of the Priors

Comparing the results of the baseline VAR(2) with those generated by VAR models with 1, 4, and 6 lags, the results do not particularly change. A higher number of lags makes the initial portion of the impulse response functions more jagged, specially for \( \gamma \) and \( r^x \), but the basic nature of the responses remains the same. Canzoneri, Cumby, and Diba also use two lags in their analysis, and in the frequentist estimates of the same VAR usually one or two lags are indicated as optimal by the usual lag length criteria.

The response of \( d \) to the \( bb \) shocks and of \( bb \) to the \( d \) shocks are very robust to changes in the relative ordering of \( bb \) and \( d \) in the Cholesky decomposition. The relative positions of \( d \) and \( bb \) do not matter for the responses of the third and fourth variable of the VAR. The relative positions of \( \gamma \) and \( r^x \) do no matter for the responses of the excess returns to the \( bb \) and \( d \) shocks either.

An important point to discuss is the selection of the parameters for the Minnesota prior and the effects of different choices of the priors on the impulse response functions. The crucial parameter to set is the relative tightness of the other variables’ lags in one equation; conditionally on this choice, the tightness of the own lags of a variable is basically irrelevant for the results.

In the VAR in levels, the non-stationarity of the system is such that, independently of the prior, one of the eigenvalues of the system is always greater than one. As a consequence, its impulse response functions are basically insensitive to the specific prior one selects. Therefore, it makes sense to make the choice of the other-variables tightness based on the behavior of the VAR de-trended \( \text{à la} \) Gourinchas and Rey, and to adopt the same parameterization also in the other versions of the model.

If, instead of \( 10^3 \), which is a very loose value, the other-variables tightness parameter is set to a very tight prior of 0.5, for instance, the estimated VAR becomes non-stationary with one eigenvalue greater than one and the predictability of the excess returns is completely lost, as Figure 13 in the set of figures of the Appendix shows. The forecasts of the VAR starting from the initial point, however, become implausible under this parameter choice. Figure 10 and 11 compare the forecasts of the two VARs, respectively for the
loose and the tight parameters. The de-trending strategy was applied in order to make the system stationary by getting rid of slow-moving time trends and, hence, it is reasonable to favor a prior more in line with this expected behavior of the data. The forecasts in Figure 11 are clearly not satisfactory in this respect. Finally, the Chi-squared test for the initial point in the two cases is basically the same and it does not reject the initial point under either of the priors. The details on these tests, the forecasts, and the eigenvalues of the VAR are presented in the Appendix.

The predictability of the excess returns disappears also for the VAR at business cycle frequencies when the tight prior is adopted, as reported in Figure 12. This discussion shows the importance of cross-equational relationships among variables for the analysis in this paper; by choosing a very loose prior on these relationships, a large power is given to the data to affect the posterior estimates.

Finally, it must be noted that under the tighter prior Gourinchas and Rey’s results would totally fail. Not only does the distinction between level and de-trended VAR disappear, but also the stabilization mechanism described for the stationary components fails. This would just reinforce, in some sense, the conclusions on the small role played by the excess returns in the sustainability of the debt.

6 Further Evidence on the Real Exchange Rate

In this Section, I investigate the possibility that the real exchange rate follows a non-Ricardian regime in responding to shocks to the trade balance, in the sense implied by the fiscal theory of price level. I will interpret these responses as pointing against a non-Ricardian behavior of the real exchange rate; it is well-known, however, that this kind of conclusion must be applied with some caution.

This kind of question is justified in this context by two observations. First, Canzoneri, Cumby, and Diba test for a non-Ricardian Regime of the level of price in the U.S. economy using a VAR framework analogous to mine. In the fiscal theory of price level, a jump of the price can correct the real government debt in response to a change in the current or expected fiscal policy, preserving the equality of the intertemporal budget constraint of the government. The transversality condition is then satisfied as an equilibrium condition for a specific realization of the price level, rather than being a constraint which must hold for any, potentially exogenous, initial price level. The real exchange rate $S_t$ is the relevant price in this open economy setup, while the foreign net debt replaces the government debt.

Second, on the empirical ground, the impulse response functions to $d$ shocks reported above usually show large impact responses of the depreciation rate, whose sign is typically different from that of the subsequent portion of the response. It seems appropriate to ask whether these large initial swings are compatible with a non-Ricardian regime for the real exchange rate, given also the fact that the dynamics of the trade balance
seems to lead to an unsustainable path of the debt.

In the intertemporal budget constraint (12), which is reported below again for convenience, the current level of the net debt inclusive of interest payments depends on the realization of the real exchange rate through the valuation effects embedded in $r^A$.

$$(1 + r_t^A) L_{t-1} - (1 + r_t^A) A_{t-1} = E_t \sum_{i=0}^{\infty} \mu_{i,t+i} D_{t+i}$$

Suppose that $D_t$ increases for some reason, the equilibrium could be re-established by a decrease of the term $(1 + r_t^A) A_{t-1}$ for example. This means a lower return rate $r_t^A$ and, therefore, an appreciation of the real exchange rate $S_t$ too. Figure 3 and 7 illustrate exactly this point for the two main cases I study.

The impact response of $\gamma$ to a $d$ shock is a depreciation and also the response at impact of $r^a$ always has the wrong sign, as can be seen from the bottom panel of Figure 5 and 9. Overall, we basically observe a depreciation of the real exchange rate that is in contradiction with the hypothesis of non-Ricardian regimes.\(^{26}\)

The contribution of $r^a$ to the determination of the level of debt, however, is only a part of the story. The overall net debt is determined by the joint action of the returns on the two assets. While on this side $r^a$ increases, pushing the net debt down, on the other side, $r^l$ responds by increasing it and the final net effect is null as shown by the response of $bb$ in Figure 3 or 7.

The responses of $bb$ have been used by Canzoneri, Cumby, and Diba to understand whether the U.S. government deficit was in a Ricardian regime. I repeat their same exercise in my setup and I actually obtain the same kind of response. The net debt $bb$ does not change at impact and it declines afterwards; if the debt is predetermined, a higher trade surplus today makes the next period’s debt necessarily decline. Following their analytical strategy, we can draw a conclusion in favor of a Ricardian regime in the relation between $bb$ and $d$ too.

Even though these results point against a non-Ricardian interpretation of the responses of the real exchange rate, this evidence must be considered far from being conclusive. There are at least two sets of problems that limit the validity of this conclusion.

First of all, the nature of the foreign debt, which is a net sum of assets and liabilities, introduces some extra complications with respect to the government debt case. In that case, for given nominal debt, the price level is the only determinant of the real debt; on the contrary, the real exchange rate is only one of the factors determining the real value of the net foreign debt. The real exchange rate is part of a broader picture in which it is also possible that $r^l$ can have a very active role, as seen in the previous section.

\(^{26}\)This time, the results from the business cycle frequencies VAR are contradictory since the real exchange rate depreciates, while $r^a$ increases. Please, note also the difference in notation between $r^A$ and $r^a$ due to the fact that (12) is defined in terms of real returns, while the VARs are estimated using the growth-adjusted returns defined in section 4.2.
Second, and most importantly, the well-known criticism to the methodology proposed by Canzoneri, Cumby, and Diba applies to this case too. In order to test for non-Ricardian regimes, we should be able to distinguish between equilibrium and off-equilibrium prices. Since we usually believe that the data we observe are only equilibrium realizations, it is quite difficult, if not impossible, to find an empirical counterpart of the theoretical off-equilibrium prices. We can treat the shocks to $d$ as off-equilibrium deficits and the responses of $\gamma$ as necessary to re-establish the correct equilibrium levels, but we would never be able to test whether the transversality condition is violated by some off-equilibrium level of the exchange rate as the theory would require.

7 Conclusions

The goal of this paper is to study the sustainability of the net foreign debt position of the U.S., in face of the rapid increase of their international debt since the beginning of the 90s.

So far, the theoretical and empirical literature that focused on the current account sustainability and the valuation effects has typically studied the external imbalances in terms of equilibrium models in which the debt sustainability was part of the set of assumptions. Contrary to this literature, I put at the center of my analysis the theoretical definition of sustainability given by the transversality condition and I provide a thorough empirical assessment of this condition and of the actual importance of the valuation channel in this respect.

A full empirical verification of the sustainability conditions is not feasible. The transversality condition and the intertemporal budget constraint involve expectations about future realization of the stochastic discount factor, the debt, and the trade balance that are difficult to model in a satisfactory way. I adopt a procedure that exploits the sample properties of the stochastic discount factor and the growth rate of the net debt to evaluate the transversality condition and I show that there are plausible statistical reasons to conclude that the American foreign debt has dangerously followed a non-sustainable trajectory over the sample 1989-2004.

Even though this is not enough to claim that the debt is unsustainable, it casts some doubts, at least, about the strength of the action of the valuation channel active during those years.

At this point, I consider the different implications of the country-wise intertemporal budget constraint of a typical stochastic two-country general equilibrium model, on one side, and a deterministic model or a linearized version of the stochastic model on the other. The two types of models differ in the way they deal with assets’ riskiness and, as a consequence, the valuation component, described by Gourinchas and Rey (2007b) for a linearized model, disappears from the budget constraint in a stochastic model.
These two different predictions are fully confirmed by the VAR analysis run on the same data set as in Gourinchas and Rey. This is an important result because it shows that the valuation effects are not systematically involved in the sustainability of the debt. They play a role in the rebalancing dynamics of the debt when data are de-trended, as shown by Gourinchas and Rey, but they are negligible for the dynamics in levels. I also show that the real exchange rate explains the excess returns movements only in part; it definitely matters for the dynamics of the returns on assets, but then the excess returns depend by a large measure on the returns on liabilities too. This is in contrast with the decomposition of the excess returns proposed by Gourinchas and Rey.

As a last step, I use the VAR setup of the paper to extend the methodology in Canzoneri, Cumby, and Diba to the case of the foreign debt in order to identify a possible non-Ricardian regime for the real exchange rate. Bearing in mind that all the flaws of this approach prevent one from taking these results as conclusive, I do not find evidence supporting this hypothesis.

My analysis clearly downplays the importance of the valuation channel for the sustainability of the net foreign debt and fully restores the traditional role of trade surpluses defined by the intertemporal approach of the current account. This paper, however, does not reject the conclusion that the valuation channel can matter for the adjustment process of the U.S. external imbalances; it rather corrects a common misinterpretation of the role of this channel, showing that relying on it to support the sustainability of the debt is a misled conception. There is an important difference between stabilization of the debt and its sustainability. It is possible that some forces help to restrain the speed at which debt is accumulated, but are not decisive in guaranteeing sustainability and this is the role I find for the valuation channel.
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In this Appendix, I provide some extra detail on the overall suitability of the estimated VAR and of the prior parameters I use in the regressions.

**Forecast and Overfitting**

Figure 10, 11, and 12 report the forecasts of the variables in the VAR for the Gourinchas and Rey’s (2007b) de-trended data and the model in levels. The baseline specification proposed does not show any evident overfitting of the series, which means that the number of lags specified in the VAR regressions is not implausibly high. I include up to six lags in the regression and the forecasts change only at the beginning of the sample as one would expect, but the quality of the fitting of the initial point expressed by the Chi-squared test declines (more below about this test). The problem of large initial transients considered by Chris Sims (1996) is not an issue here. Therefore, I can exclude that the root greater than one that characterizes the VAR in levels and the tight-prior version of the de-trended model is artificially generated by the estimation procedure. The Minnesota prior is enough to correct for it. At the end, the final choice of two lags seems
Figure 11: Initial point forecasts for the VAR with Gourinchas and Rey’s de-trending and the tight prior

Notes: Solid line actual, dotted line forecast. Bands Intervals correspond to the 14/86th percentiles.

Figure 12: Initial point forecasts for the VAR in Levels and the loose prior

Notes: Solid line actual, dotted line forecast. Bands Intervals correspond to the 14/86th percentiles.
Figure 13: Response functions to a \(bb\) unit shock. Gourinchas and Rey’s de-trending and tight prior

*Notes:* Bands Intervals correspond to the 14/86th and 5/95th percentiles. Years from the impulse on the \(x\)-axis.

Figure 14: Response functions to a \(bb\) unit shock. Business Cycle frequencies VAR and tight prior

*Notes:* Bands Intervals correspond to the 14/86th and 5/95th percentiles. Years from the impulse on the \(x\)-axis.
reasonable. The frequentist estimates of the same VAR indicate that one or two lags are preferable and also Canzoneri, Cumby, Diba include two lags in the VAR regressions of their study of the American government debt.

The differences between the forecasts in Figure 10 for the Gourinchas and Rey’s de-trended data VAR with a loose prior for the tightness parameter of the lags of the other variables and those for the VAR with a tight prior in Figure 11 justify the choice in favor of the loose prior. It is clear from the comparison of the top rows of the two figures that a tight prior significantly compromises the forecasts of $d$ and $bb$. Only the parameter of the other-variables lags is crucial for the outcome of the regression. If the prior of the own-lag parameter is set to a very loose value, while keeping the other tight, the regression’s outcome would be the same as in the case of two tight priors. I interpret the fact that the data reject the non-stationarity of the overall system when only the prior on the other-variables parameter is diffuse as an indication in favor of the specification I choose because this is the spirit of the data transformation proposed by Gourinchas and Rey. The choice of the prior is not relevant for the VAR in levels.

However, this choice of the prior does have an important effect on the impulse response functions of the excess returns, as shown in Figure 13 and 14. The responses of $r^x$ to the $bb$ shocks when the tighter prior is used are totally flat and absolutely non-significant at any horizon. They become very similar to the response for the VAR in levels. This option for the priors erases the difference between the model in levels and the de-trended variables VAR, but, in some sense, it would make the conclusions about the valuation channel even stronger. The excess returns would not be predictable even with the Gourinchas and Rey’s de-trended data and the valuation channel would lack any real importance.

**Eigenvalues and Chi-squared Tests**

From the companion form of the VAR under different specifications, I compute the eigenvalues of the system. The VAR in levels has always one eigenvalue greater than 1 and the second largest eigenvalue definitely below 1. For the baseline prior the largest eigenvalue is 1.013 (with the second largest equal to 0.92); for the tighter prior the largest is 1.018 (0.93 for the second). The baseline specification for the VAR de-trended à la Gourinchas and Rey gives a stationary system in which the largest eigenvalue is 0.98, as already explained in the main text of the paper the tighter prior determines a non-stationary system with largest eigenvalue 1.02 (second largest 0.93). The business cycle version of the model is always stationary.

From the companion form it is possible to reformulate the VAR model in the Jordan form by applying the Jordan decomposition of the companion matrix of the VAR. The new system is defined in a new set of variables, which are the combinations of the variables in the companion form of the VAR (with weights given by the elements of the left eigenvectors of the companion matrix). In the Jordan form, the system is reformulated as a set of 8 independent equations since there are 4 variables and 2 lags in the original VAR.
Let \( Z_t \) be the vector of the new variables in the Jordan form and \( \Omega \) its covariance matrix. The new variables in \( Z_t \) can be used to construct a Chi-squared test for the plausibility of the initial conditions of the data in relation to the distribution of \( Z_t \) implied by the posterior estimates of the VAR model. In fact, the statistic \( \tilde{Z}_t' \Omega \tilde{Z}_t \) (where \( \tilde{Z}_t \) is the deviation of \( Z_t \) from its stationary mean implied by the VAR estimates) must have a Chi-squared distribution with \( n \) degree of freedom, where \( n \) is the number of elements in \( Z_t \).

If the system has a non-stationary eigenvalue, the variance and mean of the corresponding element of \( Z_t \) are not well-defined any more. However, we can still construct the Chi-squared test for the remaining \( n - 1 \) stationary components of \( Z_t \). If the test statistic evaluated at the initial point is too large, then we can conclude that the initial point is at odds with the distribution implied by the VAR estimates. At that point, one can decide how to use this information in order to evaluate the plausibility of the posterior estimates of the model in relation to the initial point. This can also be used in order to compare different models. Models with smaller \( \tilde{Z}_0' \Omega \tilde{Z}_0 \) may be preferred to models with very large statistics.

The performance of the VAR in this paper is almost the same for all the models I consider. The statistic for the VAR in levels is respectively 3.1 and 2.53 for the competing specifications of the prior. It is 2.97 and 2.43 for the VAR with the Gourinchas and Rey’s de-trended data; and 3.38 and 2.5 for the VAR at business cycles. The degrees of freedom are 7 for the non-stationary systems, which have only one explosive eigenvalue, and 8 for the stationary ones. All the models report statistics well below the critical values of significance\(^{27}\) and, even though the tighter prior seems to do slightly better in all the cases, there is basically no notable difference between the two specifications in this respect. Therefore, the choice of the prior is mainly led by the considerations about the forecast properties of the de-trended model.

\(^{27}\)The 95 percent critical value is 14 for a Chi-squared distribution with 7 degrees of freedom and 15.5 for the distribution with 8 degrees.