The Dynamics of Subsidy Removal with Heterogeneous Firms*

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Abstract

We develop a dynamic general equilibrium model with heterogeneous firms and endogenous entry and exit to quantify the short- and long-term effects of eliminating operating, output, intermediate input, and interest rate subsidies. We consider the effects on welfare, production decisions, and especially measured TFP. By comparing the results to those from a model with a representative firm, we decompose TFP gains into those from reallocation of resources from low- to high-productivity firms and those from adjustment in firms’ scales of operation and mixes of inputs. Taking the entire equilibrium path into account is important because steady-state analysis alone may imply that the welfare gains from subsidy removal are understated or even negative.

Keywords: Policy distortions, Firm heterogeneity, Entry and exit, Productivity

JEL: O4, L0, L2, L5

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1 Introduction

The allocation of resources across firms is a key factor in understanding measured total factor productivity (TFP). Given the importance of TFP in accounting for macroeconomic fluctuations, it is crucial to understand how government policies affect this residual over time (see, for example, Kehoe and Prescott (2007)). In almost all economies there are at least some sectors in which low-productivity establishments are propped up by government subsidies. The objective of this study is to quantify the dynamic effects of removing subsidies commonly offered by governments—including operating, output, intermediate input, and interest rate subsidies—in a growth model with heterogeneous firms.

We are particularly concerned with how subsidy removal affects social welfare and TFP. Steady-state analysis may lead one to conclude that eliminating some subsidies lowers social welfare, when in fact the opposite is the case. We therefore consider how subsidy removal affects not only the steady state of the economy but also the entire equilibrium transition path. With respect to TFP, we carefully specify how to measure TFP in the model as it is measured in the data, and we compare this measure of TFP to a theoretical measure. The data-based measure of TFP better captures the effects of subsidy removal. By contrasting the effects of subsidy removal in our model with heterogeneous firms to a standard model with a representative firm, we show that modeling heterogeneous firms is important for capturing the full effect of subsidy removal on TFP because it allows for intrasectoral reallocation across firms. For example, lump-sum operating subsidies are non-distortionary in a model with a representative firm, but can be highly distortionary in a model with heterogeneous firms.
To model heterogeneous firms, we take a one-sector neoclassical growth model and incorporate the competitive industrial organization theory of Hopenhayn (1992), but in a general equilibrium setting, as in Hopenhayn and Rogerson (1993) (though their application is employment distortions). In the model, firms are subject to productivity shocks and make endogenous entry, operating, and exit decisions. Each firm can operate a technology with decreasing returns to scale, but there is a fixed cost of operating, so low-productivity firms may choose to exit rather than operate. Subsidies therefore affect both the intratemporal and the intertemporal (operating) decisions of firms. The model also has a representative consumer that makes a consumption–savings decision and a labor–leisure decision each period. The consumer’s savings are deposited in a financial intermediary, or bank, that invests in capital and new firms. The government offers subsidies and finances them through a lump-sum tax on the consumer (we do not incorporate distortionary taxes so as to isolate the distortionary effects of the subsidies).

We use the model to conduct a number of policy experiments. Computationally, for each experiment we calculate the two steady states and then the equilibrium transition path between them. After it is announced that there will be a policy change, there is a transition period before the policy is implemented. The shorter is the interval between announcement and implementation, the larger is the welfare gain and the smaller is the productivity gain. For each of the subsidies considered here, removal results in productivity decreases in the short run but substantial productivity increases in the long run.

In order to directly compare the operating, output, and intermediate input subsidies, we consider two cases. In one case, the subsidy levels are set so that the share of low-productivity firms that choose to operate is the same. In the other case,
the subsidy levels are set so that government expenditure as a share of GDP is the same. With equal operating thresholds, the intermediate input subsidy is the most distortive in terms of welfare and measured TFP. By contrast, with equal expenditure shares, the lump-sum operating subsidy is the most distortive. With respect to the interest rate subsidy, we find that, although it encourages overaccumulation of capital, it does not have a substantial effect on firms’ operating decisions.

This paper contributes to the literature on the link between factor misallocation and cross-country differences in productivity. The following are some papers that feature dynamic models with heterogeneous producers and idiosyncratic policy distortions (typically low-productivity producers are favored to the detriment of high-productivity producers). With these policies, firms face different after-tax prices for goods, which results in a misallocation of resources across establishments. Guner et al. (2008) analyze a model in which the government promotes small firms at the expense of large firms. Restuccia and Rogerson (2008) consider idiosyncratic taxes on firms. If the taxes are correlated with firm size, the aggregate distortions are large. Fattal (2012) emphasizes the role of endogenous entry and exit of firms. Allowing for endogenous entry and exit substantially decreases the welfare gains from removing distortionary policies. Da Rocha and Pujolas (2011) analyze the effects of idiosyncratic shocks and use Brownian motion to obtain an endogenous distribution of plants. Their model shows that failure to account for firm-level idiosyncratic shocks and endogenous exit results in overestimation of the TFP gain when policy distortions are removed. Buera et al. (2013) develop a model in which heterogeneous individuals each choose whether to be an entrepreneur or a worker. Entrepreneurs are initially subsidized, which leads to positive short-run effects when they have high productivity. As entrepreneurs lose their competitive edge, however,
they remain propped up by subsidized capital. This causes substantial long-run drops in output and productivity.

The model developed in this paper is also related to work by Chu (2003) and Gourio and Miao (2011). Chu (2003) analyzes the effects of import-substitution policies using a two-sector model. The industrial sector is subsidized at the expense of the agricultural sector. The subsidies support low-productivity industrial firms and result in economy-wide inefficiencies. In the long run, import substitution policies drive down GDP and have high welfare costs. Gourio and Miao (2011) develop a dynamic model with heterogenous firms to analyze equilibrium transition paths in response to cuts in dividend and capital gains taxes.

We differ from the above papers along various dimensions. First, we measure TFP in our model as it is typically measured in the data, which makes our analysis directly applicable to the many studies showing the importance of measured TFP in accounting for macroeconomic fluctuations. Second, we directly compare the distortive nature of output, intermediate input, and operating subsidies along several dimensions. Third, we consider the impact of removing an interest rate subsidy, which distorts intertemporal decisions. Fourth, while some papers consider only steady-state analysis, we compute equilibrium transition paths. Fifth, most papers emphasize idiosyncratic distortions to firms’ behavior, but we consider only subsidies common to all firms.

The next section presents the model and defines an equilibrium. Section 3 specifies the measurement of important statistics, including welfare, real GDP, and TFP, and specifies a growth accounting procedure. Section 4 outlines the calibration of the model. Section 5 presents the results for the policy experiments, compares the results from our model with heterogeneous firms to a similar model with a
representative firm and analyzes the transition dynamics. Section 6 concludes the paper.

2 Model

We develop a competitive dynamic general equilibrium model with a continuum of single-plant firms that make endogenous entry, operating, and exit decisions. The firms produce a homogeneous good that serves as the numeraire. The model also has a representative consumer, a bank that takes the savings of the consumer and invests in capital and firms, and a government that offers subsidies and finances them with lump-sum taxes.

2.1 Consumer

There is a representative consumer who is endowed with \(L\) units of time available for market work in each period \(t\), \(t = 0, 1, \ldots\), and amount \(\tilde{A}\) of initial assets (the amount of initial assets is specified in Subsection 2.3). The consumer derives utility from consumption and leisure according to the lifetime utility function

\[
\sum_{t=0}^{\infty} \beta^t u(C_t, \bar{L} - L^s_t),
\]

(1)

where \(\beta \in (0, 1)\) is the discount factor, \(u(\cdot, \cdot)\) is a standard period utility function, \(C_t\) is consumption, \(L^s_t\) is labor supply, and \(\bar{L} - L^s_t\) is leisure. The consumer allocates after-tax income to consumption and saving. The government subsidizes interest income at rate \(\sigma_i\). The consumer’s after-tax income is equal to labor and interest income less taxes. The consumer’s budget constraint is

\[
C_t + A_{t+1} = w_t L^s_t + (1 + (1 + \sigma_i) i_t) A_t - T_t,
\]

(2)
where $A_{t+1}$ is saving, $w_t$ is the wage rate, $i_t$ is the interest rate, and $T_t$ is the lump-sum tax.

From the consumer’s problem we can derive the intratemporal and intertemporal first-order conditions:

$$\frac{u_L(C_t, \bar{L} - L^{s}_t)}{u_C(C_t, \bar{L} - L^{s}_t)} = w_t$$  (3)

$$\frac{u_C(C_t, \bar{L} - L^{s}_t)}{\beta u_C(C_{t+1}, \bar{L} - L^{s}_{t+1})} = 1 + (1 + \sigma_t^i) i_t.$$  (4)

### 2.2 Firms

There is a continuum of heterogeneous firms that differ in productivity. The firms competitively produce a homogeneous good and are subject to idiosyncratic productivity shocks.

#### 2.2.1 Static Firm Decisions

A firm with productivity $z$ in period $t$ has the technology

$$y_t(z) = z^{1-\nu} F(k_t(z), l_t(z), x_t(z))^{\nu},$$  (5)

where $y_t(z)$ is output, $F(\cdot, \cdot, \cdot)$ is a standard production function with constant returns to scale, $k_t(z)$ is capital, $l_t(z)$ is labor, and $x_t(z)$ is intermediate inputs. Here $\nu \in (0, 1)$ so that individual firms face decreasing returns to scale. The government gives each firm a lump-sum operating subsidy of $\sigma_t^o$, an output subsidy of $\sigma_t^y$, and an intermediate input subsidy of $\sigma_t^x$. The fixed cost of operating is $f_o$ units of output. The firm’s profits are

$$\pi_t(z) = (1 + \sigma_t^y) y_t(z) - r_t k_t(z) - w_t l_t(z) - (1 - \sigma_t^x) x_t(z) - f_o + \sigma_t^o,$$  (6)

where $r_t$ is the rental rate of capital.
2.2.2 Dynamic Firm Decisions

In each period, a firm can pay the fixed operating cost and operate or exit forever. This makes each firm’s operating decision dynamic. A firm’s productivity evolves according to the Markov transition function $q(z, z')$. The present value ($v_t(z)$) of a firm with productivity $z$ in period $t$ is then defined recursively as

$$v_t(z) = \max \left\{ \pi_t(z) + \frac{1}{1 + i_{t+1}} \sum_{z'} v_{t+1}(z') q(z, z'), 0 \right\}. \quad (7)$$

Firms discount future profits according to the market rate of interest.

In the aggregate, the operating decisions of firms constitute a mixed complementarity problem. Let $\iota_t(z)$ denote the share of firms with productivity $z$ that operate in period $t$. Then the solution to the mixed complementarity problem satisfies

$$\iota_t(z) \in \begin{cases} 
\{1\} & \text{if } \pi_t(z) + \frac{1}{1 + i_{t+1}} \sum_{z'} v_{t+1}(z') q(z, z') > 0 \\
[0, 1] & \text{if } \pi_t(z) + \frac{1}{1 + i_{t+1}} \sum_{z'} v_{t+1}(z') q(z, z') = 0 \\
\{0\} & \text{if } \pi_t(z) + \frac{1}{1 + i_{t+1}} \sum_{z'} v_{t+1}(z') q(z, z') < 0 \end{cases}. \quad (8)$$

Firms with a continuation value of zero are indifferent between exiting and continuing to operate, so the share that operates is determined in equilibrium.

In period $t$, the bank finances the entry of measure $E_t$ of new firms. The cost of entry is $f_e$ units of labor. A firm that pays the cost of entry in period $t$ receives a productivity draw from probability distribution $g(z)$ and can choose to start operating in period $t + 1$. Let $m_t(z)$ denote the measure of firms with productivity $z$ in period $t$ before operating decisions have been made. The distribution of firms by productivity then evolves according to

$$m_{t+1}(z) = \sum_{\zeta} \iota_t(\zeta) m_t(\zeta) q(\zeta, z) + E_t g(z). \quad (9)$$
The measure of firms with productivity shock \( z \) in period \( t + 1 \) is the measure of incumbent firms that find it optimal to operate and receive shock \( z \) plus the measure of entrants with productivity draw \( z \).

### 2.2.3 Aggregation

We define the aggregates for output \( Y_t \), capital \( K_t \), labor used for production \( L_t^p \), intermediate inputs \( X_t \), measure of operating firms \( M_t \), and profits \( \Pi_t \) as the sum of the firm-level variables over the measure of firms that are operating:

\[
Y_t = \sum_z y_t(z) \iota_t(z) m_t(z) \\
K_t = \sum_z k_t(z) \iota_t(z) m_t(z) \\
L_t^p = \sum_z l_t(z) \iota_t(z) m_t(z) \\
X_t = \sum_z x_t(z) \iota_t(z) m_t(z) \\
M_t = \sum_z \iota_t(z) m_t(z) \\
\Pi_t = \sum_z \pi_t(z) \iota_t(z) m_t(z). \tag{15}
\]

### 2.3 Bank

There is a bank that receives deposits from the representative consumer and purchases assets of capital and ownership of firms. The timing is as follows. At the end of period \( t \), the bank receives deposits of \( A_{t+1} \) from the consumer and purchases capital and ownership of firms. In period \( t + 1 \), the bank rents the capital to firms and receives the profits of firms as dividends. At the end of period \( t + 1 \), the bank returns the deposits with interest to the consumer. The bank’s present discounted value of profits is

\[
\sum_{t=0}^{\infty} p_t \left( r_t K_t + \sum_z \pi_t(z) \iota_t(z) m_t(z) + A_{t+1} - (1 + i_t) A_t - I_t - w_t f_t E_t \right), \tag{16}
\]
where $I_t$ is investment in capital and $p_t$ is the bank’s discount factor for period $t$. Given $p_0 = 1$, the bank’s discount factors can be specified recursively according to

$$p_{t+1} = \frac{p_t}{1 + i_{t+1}}. \quad (17)$$

The bank maximizes the present discounted value of profits (16) subject to the following constraints. The law of motion of the distribution of firms (9) must hold. The law of motion of capital must hold:

$$K_{t+1} = (1 - \delta) K_t + I_t, \quad (18)$$

where $\delta$ is the depreciation rate. The bank’s assets must equal its liabilities:

$$K_t + \frac{1}{1 + i_t} \sum_z v_t(z) m_t(z) = A_t. \quad (19)$$

Finally, initial conditions on the capital stock and the measure of firms must hold:

$$K_0 = \bar{K}_0 \quad (20)$$

$$m_0(z) = \bar{m}_0(z). \quad (21)$$

Here $\bar{K}_0$ is the consumer’s initial endowment of capital and $\bar{m}_0(z)$ is the consumer’s initial endowment of firms, so the value consumer’s endowment of initial assets is

$$\bar{A}_0 = \bar{K}_0 + \frac{1}{1 + i_0} \sum_z v_0(z) \bar{m}_0(z). \quad (22)$$

From the bank’s problem we can derive two no-arbitrage conditions. The no-arbitrage condition for investment in capital is

$$r_{t+1} - \delta \leq i_{t+1}, \quad \text{if } I_t > 0. \quad (23)$$

The no-arbitrage condition for investment in new firms is

$$\frac{1}{1 + i_{t+1}} \sum_z v_{t+1}(z) g(z) \leq w_{t+1} f, \quad \text{if } E_t > 0. \quad (24)$$
The bank earns zero profits each period.

2.4 Government
The government collects lump-sum taxes from the consumer and offers operating, output, and intermediate input subsidies to producers and an interest rate subsidy to the consumer. The government’s budget constraint is

\[ \sigma_t^o M_t + \sigma_t^y Y_t + \sigma_t^r X_t + \sigma_t^i i_t A_t = T_t. \]  

(25)

2.5 Market-Clearing Conditions
Output can be used for consumption, for investment, as an intermediate good, and for payment of fixed costs of operating. Clearing in the goods market requires that

\[ C_t + I_t + X_t + f_o M_t = Y_t. \]  

(26)

Labor can be used as an input to production and for payment of fixed costs of entry. Clearing in the labor market requires that

\[ L_t^p + f_c E_t = L_t^*. \]  

(27)

2.6 Equilibrium
A competitive equilibrium consists of sequences of aggregates \( \{ \hat{Y}_t, \hat{L}_t^*, \hat{L}_t^p, \hat{C}_t, \hat{K}_t, \hat{I}_t, \hat{A}_t, \hat{M}_t \} \), prices \( \{ \hat{w}_t, \hat{r}_t, \hat{i}_t, \hat{p}_t \} \), and firm decision rules \( \{ \hat{k}_t(z), \hat{l}_t(z), \hat{x}_t(z), \hat{y}_t(z), \hat{\alpha}_t(z), \hat{\nu}_t(z), \hat{m}_t(z), \hat{\iota}_t(z) \} \) such that:

1. Given \( \{ \hat{w}_t, \hat{i}_t, \hat{T}_t \} \), the representative consumer chooses \( \{ \hat{C}_t, \hat{L}_t^*, \hat{A}_t \} \) to maximize utility (1) subject to the budget constraint (2), the initial condition on assets \( A_0 = \bar{A}_0 \) (where \( \bar{A}_0 \) satisfies (22)), and a condition to rule out Ponzi schemes.
2. Given \( \hat{w}_t \) and \( \hat{r}_t \), a firm with efficiency \( z \) in period \( t \) chooses \( \hat{k}_t(z) \), \( \hat{l}_t(z) \), \( \hat{x}_t(z) \), and \( \hat{y}_t(z) \) to maximize profits (6) subject to the technology constraint (5). Profits \( \{\hat{\pi}_t(z)\} \) satisfy (6), the value functions \( \{\hat{v}_t(z)\} \) satisfy (7), and operating decisions \( \{\hat{i}_t(z)\} \) satisfy (8).

3. Given \( \{\hat{A}_t, \hat{i}_t, \hat{v}_t(z), \hat{\pi}_t(z), \hat{p}_t\} \), the bank chooses \( \{\hat{K}_t, \hat{I}_t, \hat{m}_t(z), \hat{E}_t\} \) to maximize profits (16) subject to the law of motion of capital (18), the balance sheet condition (19), the law of motion of the distribution of firms (9), and the initial conditions (20) and (21). The bank’s sequence of discount factors \( \{\hat{p}_t\} \) satisfies (17).

4. The aggregation conditions (10)–(15) hold.

5. The government’s budget constraints (25) hold.

6. The market-clearing conditions for goods (26) and labor (27) hold.

### 3 Measurement
Here we specify the measurement of social welfare, GDP, and TFP. We also show how to perform growth accounting on real GDP.

#### 3.1 Social Welfare
We consider two measures of social welfare. The first measure of social welfare is lifetime real income. To obtain a real income index for each period, let \( \tilde{u}(\cdot, \cdot) \) be a monotonic transformation of the period utility function \( u(\cdot, \cdot) \) so as to make it homogeneous of degree one. Then lifetime real income is given by

\[
LRI = \sum_{t=0}^{\infty} \beta^t \tilde{u}(C_t, \bar{L} - L^*_t).
\] (28)
The second measure of social welfare is lifetime consumption equivalents. Suppose that we want to calculate the change in welfare in going from allocation \( \{\hat{C}_t, \hat{L}_t^s\} \) to allocation \( \{\tilde{C}_t, \tilde{L}_t^s\} \). Let \( g \) be the factor change in welfare in terms of lifetime consumption equivalents. Then \( g \) is the solution to

\[
\sum_{t=0}^{\infty} \beta^t u(g\hat{C}_t, \hat{L} - \hat{L}_t^s) = \sum_{t=0}^{\infty} \beta^t u(\tilde{C}_t, \tilde{L} - \tilde{L}_t^s). \tag{29}
\]

### 3.2 GDP

GDP can be measured in three different, yet equivalent, ways: the expenditure approach, the product approach, and the income approach. We consider each below.

An issue that arises in the calculation of GDP is how to treat investment in new firms. Creation of a new firm that can start operating in period \( t + 1 \) requires an investment of \( w_t f_{e_t} \) in period \( t \). Some of this may be investment in fixed capital, which is counted in GDP, while some of this may be investment in intangible capital, which is not. Rather than making the extreme assumption that investment in new firms is all of one type, we let \( \theta \) denote the fraction that is investment in fixed capital.

The expenditure approach to GDP is the sum of consumption, investment in capital, and fraction \( \theta \) of investment in new firms:

\[
GDP_t = C_t + I_t + \theta w_t f_{e_t} E_t. \tag{30}
\]

The product approach to GDP is gross output minus intermediate inputs:

\[
GDP_t = Y_t + \theta w_t f_{e_t} E_t - X_t - f_{o_t} M_t. \tag{31}
\]
The income approach to GDP is the sum of wages, rental income, and profits, less subsidies to business:

\[ GDP_t = w_t L_t + r_t K_t + (\Pi_t - (1 - \theta) w_t f_e E_t) - (\sigma_t^Y Y_t + \sigma_t^X X_t + \sigma_t^M M_t). \]  

(32)

Notice that investments in intangible capital are expensed by firms and thus subtracted from profits. Using the equilibrium conditions, it is straightforward to verify that the three measures of GDP are equal.

Now that we have specified nominal GDP, we specify how to obtain real GDP. Consistent with most national accounts data, we use a chain-weighted GDP deflator. This involves calculating a Fisher price index, which is the geometric mean of the Paasche and Laspeyres price indexes. Letting \( P_0 = 1 \), we use expenditures to recursively calculate the chain-weighted GDP deflator as

\[ P_{t+1} = \left( \frac{C_{t+1} + I_{t+1} + \theta w_{t+1} f_e E_{t+1}}{C_{t+1} + I_{t+1} + \theta w_t f_e E_t} \right)^{1/2} \left( \frac{C_t + I_t + \theta w_{t+1} f_e E_t}{C_t + I_t + \theta w_t f_e E_t} \right)^{1/2} P_t. \]  

(33)

Notice that the relative price of investment in new firms is the wage. If \( \theta = 0 \), then the price index is constant at one. If \( \theta \in (0, 1] \), then the price index varies over the transition path with the wage. Real GDP is

\[ RGD P_t = \frac{GD P_t}{P_t}. \]  

(34)

### 3.3 TFP

We consider two measures of TFP: *measured* and *theoretical*. With our measure of real GDP, we can approach the calculation of measured TFP in the model as a macroeconomist typically would using data. We assume a standard Cobb–Douglas form for the aggregate production function and calculate measured TFP as the residual

\[ TFP_t = \frac{RGD P_t}{K_t^\alpha L_t^{1-\alpha}}, \]  

(35)
where \( \tilde{K}_t \) is the measured input of fixed capital, \( \tilde{L}_t \) is the measured input of labor, and \( 1 - \alpha \) is labor’s share of income. In the context of this model, and given our assumptions on the role of fixed capital in the creation of new firms, we have measured inputs of

\[
\tilde{K}_t = K_t + \frac{\theta}{1 + \gamma_t} \sum_z v_t(z) m_t(z) \tag{36}
\]

\[
\tilde{L}_t = L_t^s. \tag{37}
\]

We set labor’s share of income according to its steady-state value with no subsidy in place:

\[
1 - \alpha = \frac{w \tilde{L}}{GDP}. \tag{38}
\]

We can also obtain a theoretical measure of TFP. It is straightforward to show that output in the model can be expressed as

\[
Y_t = Z_t G(K_t, L_t^p, X_t, M_t),
\]

where

\[
Z_t = \left( \frac{1}{M_t} \sum_z zt_t(z) m_t(z) \right)^{1-\nu} \tag{39}
\]

and

\[
G(K, L, X, M) = F(K, L, X)^{\nu} M^{1-\nu}. \tag{40}
\]

Theoretical TFP, given by \( Z_t \), is a weighted average of operating firms’ productivities, while \( G(\cdot, \cdot, \cdot, \cdot) \) is an aggregate production function with constant returns to scale. Even though individual firms face decreasing returns to scale, there are constant returns to scale in the aggregate once the measure of operating firms is taken into account.
By contrasting the two types of TFP, it is clear that measured TFP is a more complete measure of productivity. Following the removal of a subsidy, theoretical TFP only reflects the changes in firms’ operating decisions. But subsidies can create other distortions, such as distortions in firms’ scales of operation and their input mixes, that are not captured by theoretical TFP but are captured by measured TFP. Measured TFP also has the advantage of being consistent with how macroeconomists typically measure TFP in the data.

3.4 Growth Accounting

We can perform growth accounting using time series from the model just as a macroeconomist would using time series from the data. We use the growth accounting method developed by Hayashi and Prescott (2002) and used extensively in Kehoe and Prescott (2007). We can express real GDP in our model by rewriting (35) as

\[ RGDP_t = TFP_t^{\frac{1}{1-\alpha}} \left( \frac{\hat{K}_t}{RGDP_t} \right)^{\frac{\alpha}{1-\alpha}} \hat{L}_t. \]  

(41)

This makes it clear that we can decompose real GDP into three factors: the TFP factor is \( TFP_t^{1/(1-\alpha)} \), the capital factor is \( (\hat{K}_t/RGDP_t)^{\alpha/(1-\alpha)} \), and the labor factor is \( \hat{L}_t \). This growth accounting method was designed so that, on a balanced growth path with constant growth of TFP, all growth is due to the TFP factor; though the capital stock is also growing, this is entirely due to the growth of TFP, so the capital factor remains constant. Our model has a steady state rather than a balanced growth path, so our growth accounting exercise is equivalent to performing growth accounting on detrended data.
Table 1: Calibration of the Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibrated Value</th>
<th>Description</th>
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<td>1.000</td>
<td>Time available</td>
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<tr>
<td>$\gamma$</td>
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<td>Consumption weight</td>
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<td>$\psi_2$</td>
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<td>Fixed entry cost</td>
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<td>$f_o$</td>
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<td>Fixed operating cost</td>
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<tr>
<td>$\delta$</td>
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<td>Depreciation rate</td>
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</table>

4 Calibration

Rather than calibrating the model to a specific country and time, we make our analysis as widely applicable as possible by using standard functional forms and parameter values. We calibrate the model to the steady state with no policy distortions. Table 1 summarizes the calibration.

We choose standard functional forms. We specify the period utility function in (1) as

$$u(C, \bar{L} - L^*) = \gamma \log C + (1 - \gamma) \log(\bar{L} - L^*),$$

where $\gamma \in (0, 1)$. We specify the constant-returns-to-scale component of the production function in (5) as

$$F(k, l, x) = k^{\psi_1} l^{\psi_2} x^{1-\psi_1-\psi_2},$$

where $\psi_1 \in (0, 1)$ and $\psi_2 \in (0, 1)$.

We choose standard parameter values. We normalize units of labor so that $\bar{L} = 1$ and set $\gamma = 0.35$ to obtain $L^*/\bar{L} = 0.3$. Assuming that 100 hours of time are available for market work each week, this implies 30 hours of market work per week. We set $\beta = 0.96$ to obtain a standard long-term real interest rate of 4
percent. Following Restuccia and Rogerson (2008), we set \( \nu = 0.85 \). We assume that intermediate inputs to production account for \( 1/2 \) of the total value of output. Then, based on standard income shares for capital and labor of \( 1/3 \) and \( 2/3 \), we set \( \psi_1 = 1/6 \) and \( \psi_2 = 1/3 \). We assume that the fixed cost of investment in a new firm is one unit of labor. We target an investment to GDP ratio of 15 percent, which implies a depreciation rate of 6.5 percent. We initially assume that \( \theta = 1 \) but relax this assumption later.

Next we specify firm productivity levels and the Markov transition function. We choose a simple but transparent specification, similar to Chu (2003). There two possible types of operating firms: low-productivity and high-productivity. High-productivity firms are at the frontier of technology, while low-productivity firms have fallen behind by a factor of one-half. There is also an absorbing state with zero productivity. Thus \( z \in \{0, 1, 2\} \). All firms enter at the frontier of technology, so

\[
g(z) = \begin{cases} 
1 & \text{if } z = 2 \\
0 & \text{otherwise} 
\end{cases}.
\]

With 10 percent probability, a high-productivity firm falls behind the technological frontier and becomes a low-productivity firm. With 15 percent probability, a low-productivity firm dies. The Markov transition matrix is

\[
q = \begin{bmatrix}
1 & 0 & 0 \\
0.15 & 0.85 & 0 \\
0 & 0.1 & 0.9 
\end{bmatrix}.
\]

We are interested in the case where low-productivity firms cannot survive without government assistance. Accordingly we set the fixed cost of operating sufficiently high at \( f_o = 0.04 \) units of output.
5 Policy Experiments

In this section, we use the calibrated model to conduct policy experiments. Not only do we analyze the steady-state effects of subsidy removal, as is typical in the literature, but we also numerically solve for equilibrium transition paths. We consider the following transition dynamics. In period 0, the economy is in a steady state. In period 1, all decision makers learn that there will be a permanent policy change in period 5. The new policy is implemented in period 5 and the economy converges to a new steady state. The algorithms for calculating the steady states and transition paths are given in Appendix 7.

We conduct a number of policy experiments. We first consider the steady-state and welfare effects of the removal of operating, output, and intermediate input subsidies. We choose the initial levels of these subsidies to make them comparable. In Subsection 5.1 we choose the initial subsidy levels so that, for each subsidy, the share of low-productivity firms that operate is the same. In Subsection 5.2 we choose the initial subsidy levels so that, for each subsidy, government expenditure is the same share of GDP. Plausible levels of the interest rate subsidy do not cause a substantial measure of low-productivity firms to operate, so we discuss this subsidy separately in Subsection 5.3. We also contrast the results of our model with heterogeneous firms to a model with a representative firm.

5.1 Subsidy Levels Based on Operating Thresholds

In this subsection we analyze the removal of operating, output, and intermediate input subsidies. We are particularly interested in levels of the subsidies at which low-productivity firms operate. To make the subsidies comparable, we set the initial level of each subsidy at the threshold where 80 percent of low-productivity
<table>
<thead>
<tr>
<th>Statistic</th>
<th>$\sigma^o = 0.0105$</th>
<th>$\sigma^y = 13.87%$</th>
<th>$\sigma^x = 26.32%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP</td>
<td>−6.5</td>
<td>−28.3</td>
<td>−27.9</td>
</tr>
<tr>
<td>Capital stock</td>
<td>−10.4</td>
<td>−44.6</td>
<td>−48.2</td>
</tr>
<tr>
<td>Labor supply</td>
<td>−4.9</td>
<td>−20.7</td>
<td>−25.9</td>
</tr>
<tr>
<td>Intermediate inputs</td>
<td>−10.4</td>
<td>−44.6</td>
<td>−61.9</td>
</tr>
<tr>
<td>Measure of entrants</td>
<td>−11.9</td>
<td>−26.1</td>
<td>−31.0</td>
</tr>
<tr>
<td>Measured TFP</td>
<td>1.5</td>
<td>4.7</td>
<td>12.6</td>
</tr>
<tr>
<td>Theoretical TFP</td>
<td>1.6</td>
<td>1.6</td>
<td>1.6</td>
</tr>
<tr>
<td>Real income index</td>
<td>−0.2</td>
<td>−0.7</td>
<td>3.8</td>
</tr>
<tr>
<td>Consumption</td>
<td>−4.7</td>
<td>−21.9</td>
<td>−18.3</td>
</tr>
<tr>
<td>Leisure</td>
<td>2.3</td>
<td>13.0</td>
<td>18.2</td>
</tr>
</tbody>
</table>

In this case, low-productivity firms are 20.1 percent of the total measure of operating firms and high-productivity firms are 79.9 percent. With this threshold, the lump-sum operating subsidy is $\sigma^o = 0.0105$, the output subsidy is $\sigma^y = 0.1387$, and the intermediate input subsidy is $\sigma^x = 0.2632$.

Table 2 summarizes the steady-state effects of removing each subsidy and Table 3 shows the effects on social welfare. There are several important points to be made about the tables. The lump-sum operating subsidy is the least distortive because it only affects firms’ operating decisions, while output and intermediate input subsidies also affect firms’ scales of operation and input mixes. The intermediate input subsidy is the most distortive (except to real GDP). All policy reforms lead to substantial drops in output and inputs. Subsidy removal also reduces the incentive to invest in new firms, so the measure of entrants decreases substantially in each case. Although Table 2 compares real GDP and measured TFP between the two steady states, it is important to note that, since $\theta = 1$ here, the price index used in the calculation of these statistics depends on the entire transition path, as (33) indicates.
Table 3: Percentage Change in Welfare, Equal Operating Thresholds

<table>
<thead>
<tr>
<th>Welfare measure</th>
<th>Policy removed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma^o = 0.0105$</td>
</tr>
<tr>
<td>Lifetime real income</td>
<td>0.5</td>
</tr>
<tr>
<td>Lifetime cons. equivs.</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Consider the results with respect to TFP. In each case, theoretical TFP increases by 1.6 percent. This is because theoretical TFP only captures changes in the weighted average productivity of operating firms and, by construction, the share of low-productivity firms that exit is the same in each case here. The effect on measured TFP, however, varies dramatically. With only a lump-sum operating subsidy, the effects of subsidy removal on measured and theoretical TFP are of similar magnitudes because, if a firm operates, the subsidy does not directly distort its scale of operation; the subsidy only directly affects a firm’s operating decision. With the other subsidies, the effects on measured TFP are much larger because they distort each firm’s scale of operation. Removal of the output subsidy increases measured TFP by 4.7 percent and removal of the intermediate input subsidy increases measured TFP by 12.6 percent.

Consider the results with respect to welfare. In the absence of distortions, the equilibrium allocation is Pareto-efficient, so the removal of any subsidy increases welfare, whether measured by lifetime real income or lifetime consumption equivalents (see Table 3). Both of these welfare measures take into account the entire equilibrium path. If we just look at the change in steady-state real income, we see that removal of the operating and output subsidies actually results in a decline in steady-state real income (see Table 2). This result highlights the importance of taking the transition into account when conducting welfare analysis.
The subsidies considered here require very different levels of government expenditure relative to GDP. Government expenditure relative to GDP is 6 percent for the operating subsidy, 29 percent for the output subsidy, and 39 percent for the intermediate input subsidy.

5.2 Subsidy Levels Based on Expenditure Share of GDP

Here we again analyze operating, output, and intermediate input subsidies, but we choose the initial subsidy levels so that subsidy expenditures as a share of GDP are the same in each experiment. If subsidy expenditures are 39.2 percent of GDP, as we assume here, then the lump-sum operating subsidy is \( \sigma^o = 0.0351 \), the output subsidy is \( \sigma^y = 0.1763 \), and the intermediate input subsidy is \( \sigma^x = 0.2632 \).

This analysis has very different results from the previous section. As Tables 4 and 5 show, when subsidy expenditures are the same share of GDP, the operating subsidy is the most distortive—in terms of measured TFP and welfare—rather than the least. On the other hand, the output subsidy has the largest effect on production decisions in the sense that the reductions in real GDP and the capital stock are larger. The decrease in steady-state consumption is also largest with the output subsidy, yet it has the smallest impact on welfare, which takes into account the entire equilibrium path.

Table 6 shows how each subsidy affects the distribution of operating firms. The lump-sum operating subsidy has the most direct effect on period profits and the value of a firm, so the initial share of low-productivity firms is highest with this subsidy. Removal of the operating subsidy thus generates the largest gains in productivity, as measured by both theoretical and measured TFP.
### Table 4: Percentage Change in Steady-State Level, Equal Exp. Shares

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Policy removed</th>
<th>( \sigma^o = 0.0351 )</th>
<th>( \sigma^y = 17.63% )</th>
<th>( \sigma^x = 26.32% )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP</td>
<td>-29.1</td>
<td>-33.8</td>
<td>-27.9</td>
<td></td>
</tr>
<tr>
<td>Capital stock</td>
<td>-39.5</td>
<td>-53.3</td>
<td>-48.2</td>
<td></td>
</tr>
<tr>
<td>Labor supply</td>
<td>-25.6</td>
<td>-25.9</td>
<td>-25.9</td>
<td></td>
</tr>
<tr>
<td>Intermediate inputs</td>
<td>-39.5</td>
<td>-53.3</td>
<td>-61.9</td>
<td></td>
</tr>
<tr>
<td>Measure of entrants</td>
<td>-56.7</td>
<td>-26.7</td>
<td>-31.0</td>
<td></td>
</tr>
<tr>
<td>Measured TFP</td>
<td>13.3</td>
<td>7.1</td>
<td>12.6</td>
<td></td>
</tr>
<tr>
<td>Theoretical TFP</td>
<td>3.4</td>
<td>2.9</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>Real income index</td>
<td>5.1</td>
<td>0.4</td>
<td>3.8</td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>-15.0</td>
<td>-25.7</td>
<td>-18.3</td>
<td></td>
</tr>
<tr>
<td>Leisure</td>
<td>17.9</td>
<td>18.2</td>
<td>18.2</td>
<td></td>
</tr>
</tbody>
</table>

### Table 5: Percentage Change in Welfare, Equal Exp. Shares

<table>
<thead>
<tr>
<th>Welfare measure</th>
<th>Policy removed</th>
<th>( \sigma^o = 0.0351 )</th>
<th>( \sigma^y = 17.63% )</th>
<th>( \sigma^x = 26.32% )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lifetime real income</td>
<td>8.8</td>
<td>4.7</td>
<td>6.1</td>
<td></td>
</tr>
<tr>
<td>Lifetime cons. equivs.</td>
<td>26.9</td>
<td>13.7</td>
<td>18.1</td>
<td></td>
</tr>
</tbody>
</table>

### Table 6: Shares of Operating Firms, Equal Exp. Shares

<table>
<thead>
<tr>
<th>Firm type</th>
<th>( \sigma^o )</th>
<th>( \sigma^y )</th>
<th>( \sigma^x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-productivity</td>
<td>0.0351</td>
<td>17.63%</td>
<td>26.32%</td>
</tr>
<tr>
<td>High-productivity</td>
<td>0.995</td>
<td>99.5%</td>
<td>99.5%</td>
</tr>
</tbody>
</table>
5.3 Interest Rate Subsidy
Here we consider the removal of an interest rate subsidy of 50 percent. The subsidy lowers the interest rate and the rental rate of capital. We find no level of this subsidy at which a substantial share of low-productivity firms choose to operate. This is because the negative effects of the subsidy on firms’ profits outweigh the positive effects. The subsidy raises firms’ profits because it lowers the rental rate of capital and, by lowering the interest rate, decreases firms’ discounting of future profits. But, by dramatically increasing the capital stock and therefore the marginal product of labor, the subsidy increases the wage and, on net, lowers firms’ profits. This does not induce more low-productivity firms to operate, so the changes in output, productivity, and welfare are due to the readjustment of firms’ inputs to their undistorted levels.

As shown in Table 7, following the reform, the steady-state comparison shows that the real interest rate increases by the amount of the subsidy (for all other subsidy scenarios, the steady-state real interest rate is unaffected). The decline of the capital stock to its undistorted steady-state level (a 21.0 percent decrease) is large relative to labor (a 3.2 percent decrease) because the subsidy directly impacts the rental rate of capital. The adjustment of capital and labor cause real GDP to decline by 9.2 percent. Steady-state real income declines by 1.2 percent. When taking the full transition path into account, however, lifetime real income and lifetime consumption equivalents increase by 0.2 and 0.5 percent respectively.

5.4 Comparison to a Representative Firm Model
In this subsection, we compare the results of our model with heterogeneous firms to a model with a representative firm. The representative firm uses the technology
Table 7: Percentage Change in Steady-State Level, Interest Rate Subsidy

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Policy removed</th>
<th>$\sigma^i = 50%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP</td>
<td></td>
<td>$-9.2$</td>
</tr>
<tr>
<td>Capital stock</td>
<td></td>
<td>$-21.0$</td>
</tr>
<tr>
<td>Labor supply</td>
<td></td>
<td>$-3.2$</td>
</tr>
<tr>
<td>Intermediate inputs</td>
<td></td>
<td>$-9.2$</td>
</tr>
<tr>
<td>Measure of entrants</td>
<td></td>
<td>$-10.1$</td>
</tr>
<tr>
<td>Measured TFP</td>
<td></td>
<td>$0.9$</td>
</tr>
<tr>
<td>Theoretical TFP</td>
<td></td>
<td>$0.0$</td>
</tr>
<tr>
<td>Real income index</td>
<td></td>
<td>$-1.2$</td>
</tr>
<tr>
<td>Consumption</td>
<td></td>
<td>$-5.9$</td>
</tr>
<tr>
<td>Leisure</td>
<td></td>
<td>$1.5$</td>
</tr>
</tbody>
</table>

given by (43). There is no firm entry or exit and there are no fixed costs of firm entry or operating. Though an operating subsidy creates substantial distortions in our model with heterogeneous firms, it is not at all distortionary in a model with a representative firm.

In what follows, we focus on the effects of removing an output subsidy of 17.63 percent. The long-run effects of removing this subsidy are summarized in Table 8. The magnitudes of the effects are similar, except with respect to TFP. Measured TFP increases by 7.1 percent in the heterogeneous firms model but only by 3.2 percent in the representative firm model. The representative firm model understates the gain in TFP by a factor of 2.2 because it only captures productivity increases from the adjustment of capital, labor, intermediate inputs, and output to their undistorted levels, whereas the heterogeneous firms model not only captures this effect but also captures productivity gains occurring through reallocation of resources from low- to high-productivity firms. Figure 1 graphs the equilibrium transition paths for both measured and theoretical TFP (see Appendix 7.3 for transition dynamics for operating, intermediate input, and interest rate subsidies).
Table 8: Percentage Change in Steady-State Level, Output Subsidy

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Heterogeneous firms</th>
<th>Representative firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP</td>
<td>-33.8</td>
<td>-35.0</td>
</tr>
<tr>
<td>Capital stock</td>
<td>-53.3</td>
<td>-54.5</td>
</tr>
<tr>
<td>Labor supply</td>
<td>-25.9</td>
<td>-26.0</td>
</tr>
<tr>
<td>Intermediate input</td>
<td>-53.3</td>
<td>-54.5</td>
</tr>
<tr>
<td>Measured TFP</td>
<td>7.1</td>
<td>3.2</td>
</tr>
<tr>
<td>Theoretical TFP</td>
<td>2.9</td>
<td>0.0</td>
</tr>
<tr>
<td>Real income index</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>Consumption</td>
<td>-25.7</td>
<td>-27.1</td>
</tr>
<tr>
<td>Leisure</td>
<td>18.2</td>
<td>18.7</td>
</tr>
</tbody>
</table>

Table 9: Percentage Change in Welfare, Output Subsidy

<table>
<thead>
<tr>
<th>Welfare measure</th>
<th>Heterogeneous firms</th>
<th>Representative firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lifetime real income</td>
<td>4.7</td>
<td>4.8</td>
</tr>
<tr>
<td>Lifetime cons. equv.</td>
<td>13.7</td>
<td>14.0</td>
</tr>
</tbody>
</table>

While productivity gains are largest for the heterogeneous firms model, TFP falls during the period leading up to the implementation of the reform. As the next subsection shows, this is largely driven by capital accumulation relative to the level of GDP after the announcement of the reform. Though the effects on productivity differ between the two models, the effects on welfare are similar, as Table 9 shows, because the effects on consumption and leisure are similar.

5.5 Transition Dynamics and Growth Accounting

Here we analyze the elimination of a 17.63 percent output subsidy on the equilibrium transition paths of the model using the growth accounting framework in Subsection 3.4. In Appendix 7.3, we show the transition dynamics resulting from removal of operating, intermediate input, and interest rate subsidies.

When informed of the subsidy removal, production inefficiencies are exacerbated in the short run because producers take advantage of the remaining subsi-
dized periods by overcapitalizing relative to real value added. Following the reform, productivity increases because producers readjust their scales of operation to undistorted levels.

Figure 2 graphs the growth accounting implied by (41). Because real GDP and the labor factor decline at roughly the same rate, the short-term TFP decline is accounted for by the short-term increase in the capital factor. Following the reform, real GDP declines slowly to its new steady-state level, while the TFP factor increases by 22.4 percent from its low point of 91.4 (or 11.8 percent above its initial steady-state level). This productivity gain is driven mainly by the rapid decline in the capital factor and the convergence of the capital–labor ratio to its undistorted steady-state level.

The length of time between when a reform is announced and when it is implemented matters. We primarily consider the case where the policy is implemented in period $t = 5$, but in Table 10 we consider two other cases. If the reform is announced and implemented in period $t = 1$, then the policy change is a complete surprise. We also consider a long period of anticipation by waiting until period $t = 15$ to implement the policy. In terms of welfare, the consumer is better off the sooner the distortions are removed (see Table 10). In terms of productivity, the gains from subsidy removal are less pronounced the shorter is the interval between the announcement and the implementation of the reform. As noted earlier, the reason why the change in TFP varies with the timing of policy announcement and implementation is that the price index used in the calculation of TFP takes the entire transition path into account.

The above analysis is performed assuming that investment in new firms is all fixed capital ($\theta = 1$). We next consider the cases where investment in new firms
Table 10: Percentage Change, Output Subsidy

<table>
<thead>
<tr>
<th>Statistic</th>
<th>$t = 1$</th>
<th>$t = 5$</th>
<th>$t = 15$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lifetime real income</td>
<td>5.3</td>
<td>4.7</td>
<td>3.3</td>
</tr>
<tr>
<td>Lifetime cons. equivs.</td>
<td>15.2</td>
<td>13.7</td>
<td>9.6</td>
</tr>
<tr>
<td>Measured TFP</td>
<td>4.9</td>
<td>7.1</td>
<td>7.2</td>
</tr>
</tbody>
</table>

is equally split between intangible and fixed capital ($\theta = 0.5$) and is all intangible capital ($\theta = 0$). The effect of the classification of investment in new firms has only small effects on the steady-state values of the equilibrium objects (not shown in the tables). As Figure 3 shows, however, this classification has sizeable effects on the steady-state value of the TFP factor and the transition paths for real GDP and the three growth factors. Steady-state comparison shows that, if some or all of the investment in new firms is intangible capital ($\theta = 0.5$ or $\theta = 0$), then the increase in the TFP factor is less pronounced (a gain of only 9.5 or 7.6 percent). As seen in both subfigures, the dynamics are similar, but more exaggerated, for the case where $\theta = 0$. Productivity increases after the announcement of the reform because the initial fall in real GDP and the over-capitalization are less pronounced. Following the reform, productivity immediately drops to below the initial steady-state level, as there is an acute increase in the capital factor and a rapid decline in real GDP. Then it converges to its new steady state level, 9.5 percent above the initial steady state with $\theta = 0.5$ and 7.6 percent above with $\theta = 0$.

Figure 4 graphs the transition dynamics, assuming that $\theta = 1$, for investment in new firms and capital in the first panel and for the wage and rental rate in the second panel (Appendix 7.3 provides transition dynamics for operating, intermediate input, and interest rate subsidies). As shown in the first panel, leading up to the reform, the bank finds it optimal to temporarily disinvest in new firms. After the
reform is implemented, investment in new firms jumps to a level above the initial steady state and then converges to its new steady state level, 50 percent below its initial level. The bank reduces the capital stock leading up to the reform. Immediately following the reform, the bank disinvests in capital. Then the capital stock converges to its new steady state, also 50 percent below its pre-reform level. Following the reform, the consumer finds it optimal to reduce labor supply and enjoy more leisure. As shown in the second panel, the contraction in labor supply causes the wage rate to converge to a new steady state level, 42 percent below its initial level. The interest rate is constant in the long run, and its equilibrium transition path mimics that of investment in capital.

6 Conclusion

We develop a simple dynamic general equilibrium model with heterogeneous firms and endogenous entry and exit. We use the calibrated model to quantitatively analyze the effects of removing operating, output, intermediate input, and interest rate subsidies. We measure important statistics, such as real GDP and TFP, so as to be consistent with the data. For capturing the full effects of subsidy removal, we find that it is important to incorporate heterogeneous firms and analyze full equilibrium paths. A caveat to our analysis is that it is less applicable to sectors in which fixed costs of operating are less important. Future research involves using the methodology here to quantitatively account for the effects of subsidy removal or imposition in particular economies and sectors.
References


Figure 1: Comparison of TFP

Figure 2: Growth Accounting, Output Subsidy Case
Figure 3: Growth Accounting, Output Subsidy, $\theta = 0.5$ (Left) and $\theta = 0$ (Right)

Figure 4: Transition Dynamics of Equilibrium Objects
7 Appendix

This appendix has three sections. Section 7.1 provides an algorithm for calculating the steady state. Section 7.2 presents an algorithm for calculating the transition path between two steady states. Each involves computing the solution to a system of nonlinear equations using Newton’s (or a similar) method. Newton’s method requires continuous first derivatives. As a result, we need an approximation of (8). We use a standard Heaviside function:

\[ \mu_t(z) = \frac{1}{2} + \frac{1}{\pi} \tan^{-1} \left( \frac{\pi_t(z) + (1/(1+i_{t+1})) \sum_{z'} v_{t+1}(z) q(z, z')}{\varepsilon} \right), \]  

where \( \varepsilon \) is small. Section 7.3 provides graphs of transition dynamics for operating, intermediate input, and interest rate subsidies.

7.1 Algorithm to Calculate the Steady State

To calculate the steady state, we use the following algorithm. (Where equations are referenced, we are referring to the steady-state versions in which \( t \) is eliminated.)

1. Solve for \( i \) using (4).

2. Solve for \( r \) using (23).

3. Guess \( L^*, C, E, \) and \( w \).

4. Solve for \( k(z), l(z), \) and \( x(z) \) using the profit-maximization conditions

\[ \nu(1 + \sigma^y)z^{1-\nu}F_k(k(z), l(z), x(z))^{\nu-1} = r \]  

\[ \nu(1 + \sigma^y)z^{1-\nu}F_l(k(z), l(z), x(z))^{\nu-1} = w \]  

\[ \nu(1 + \sigma^y)z^{1-\nu}F_x(k(z), l(z), x(z))^{\nu-1} = 1 - \sigma^x. \]

5. Calculate \( y(z) \) using (5).
6. Calculate \( \pi(z) \) using (6).

7. Use value function iteration to solve for \( v(z) \), as given by (7).

8. Calculate \( \iota(z) \) using (46).

9. Solve for \( m(z) \) using (9).

10. Calculate \( Y, L, K, X, \Pi, \) and \( M \) using (10)–(15).

11. Calculate \( A \) using (19).

12. Calculate \( T \) using (25).

13. Solve for \( I \) using (18).

14. If (3), (24), (26), and (27) hold, stop. If not, go back to step 3.

By Walras’s law, (2) also holds approximately.

### 7.2 Algorithm to Calculate the Transition Dynamics

To calculate the equilibrium transition path between two steady states, we use the following algorithm. We assume that the economy is in the initial steady state in period 0 and in the final steady state in period \( T + 1 \), where \( T \) is large. Because the non-negativity constraints may bind, we approximate the no-arbitrage conditions (23) and (24) by

\[
 r_{t+1} - \delta + \eta \min \{ I_t, 0 \}^2 = i_{t+1} \\
 \frac{1}{1 + i_{t+1}} \sum_z v_{t+1}(z) g(z) + \eta \min \{ E_t, 0 \}^2 = w_t f_e, 
\]

where \( \eta \) is large. We take the values of \( K_1, A_1, m_1(z), r_1, \) and \( i_1 \) from the initial steady state and any values from periods \( t = T + 1, T + 2, \ldots \) from the final steady state.
1. Guess \( \{L^*_t, C_t, E_t, w_t\}^T_{t=1} \) and \( \{K_t, r_t\}^T_{t=2} \).

2. Solve for \( \{i_t\}^T_{t=2} \) using (50).

3. Solve for \( \{k_t(z), l_t(z), x_t(z)\}^T_{t=1} \) using the profit-maximization conditions

\[
\nu(1 + \sigma_t^y)z^{1-\nu} F_k(k_t(z), l_t(z), x_t(z))^{\nu-1} = r_t \tag{52}
\]

\[
\nu(1 + \sigma_t^y)z^{1-\nu} F_l(k_t(z), l_t(z), x_t(z))^{\nu-1} = w_t \tag{53}
\]

\[
\nu(1 + \sigma_t^y)z^{1-\nu} F_x(k_t(z), l_t(z), x_t(z))^{\nu-1} = 1 - \sigma_t^x. \tag{54}
\]

4. Calculate \( \{y_t(z)\}^T_{t=1} \) using (5).

5. Calculate \( \{\pi_t(z)\}^T_{t=1} \) using (6).

6. Solve for \( \{v_t(z)\}^T_{t=1} \) using backward induction and (7).

7. Calculate \( \{\ell_t(z)\}^T_{t=1} \) using (46).

8. Calculate \( \{m_t(z)\}^T_{t=2} \) using (9).

9. Calculate \( \{Y_t, L^*_t, X_t, \Pi_t, M_t\}^T_{t=1} \) using (10) and (12)–(15).

10. Calculate \( \{A_t\}^\infty_{t=2} \) using (19).

11. Calculate \( \{T_t\}^T_{t=1} \) using (25).

12. If (3), (51), (26), and (27) hold for \( t = 1, 2, ..., T \), (11) holds for \( t = 2, 3, ..., T \), and (4) holds for \( t = 1, 2, ..., T - 1 \), stop. If not, go back to step 1.

By Walras’s law, (2) also holds approximately.

### 7.3 Transition Dynamics for Other Subsidies

Here we show transition dynamics for an operating subsidy of \( \sigma^o = 0.0351 \), an intermediate input subsidy of \( \sigma^x = 0.2632 \), and an interest rate subsidy of \( \sigma^i = 0.5 \).
Figure 5: Comparison of TFP, $\sigma^o$

Figure 6: Growth Accounting, $\sigma^o$
Figure 7: Transition Dynamics of Equilibrium Objects, $\sigma^o$

Figure 8: Comparison TFP, $\sigma^x$
Figure 9: Growth Accounting, $\sigma^x$

![Growth Accounting Graph]

Figure 10: Transition Dynamics of Equilibrium Objects, $\sigma^x$

![Transition Dynamics Graph]
Figure 11: TFP Comparison, $\sigma^i$

Figure 12: Growth Accounting, $\sigma^i$
Figure 13: Transition Dynamics of Equilibrium Objects, $\sigma^i$