

The Private Memory of Aggregate Shocks

Carlos E. da Costa, *Fundação Getulio Vargas*

Vitor F. Luz, *Yale University*

[Preliminary and Incomplete]

Abstract

Using two period versions of both a Atkeson and Lucas (1992) preference shock model and a dynamic Mirrlees (1971) economy we show that constrained optimal allocations display memory with respect to *aggregate* that depends on the degree of persistence of private shocks: the more persistent the shock, the less memory allocations display. We also investigate some macroeconomic consequences of private information. In doing so we move on from a normative to a positive view of our results. Our results indicate that neither the consequences for asset pricing nor for the behavior of the labor wedge adhere to the stylized facts for these two variables.

1. Introduction

Following Wilson (1968)'s landmark contribution, much have been learnt about optimal risk sharing. In an environment in which all idiosyncratic shocks are observed, the mutualization of private risks and its consequences for the dynamics of private consumption as a function of aggregate consumption are now well understood.

When there is private information regarding idiosyncratic shocks, the early contribution of Rogerson (1985) followed by the methodological advances found in Green (1987), Spear and Srivastava (1987) and Thomas and Worrall (1990) allowed a better understanding of fundamental issues of dynamic insurance schemes. Contrary to the case with no private information, (constrained) optimal schemes exhibit memory, leading to extreme results in the long run.¹ Most of this latter literature, however, focuses on the case in which only private risk exists, making assumptions on the nature of idiosyncratic shocks that ultimately lead to the elimination of aggregate risk, Phelan (1994) being a noteworthy exception.

¹The non-existence of non-degenerate steady-states and the associated immiseration results are now well understood consequences of back loading incentives that typically characterizes such environments.

In this paper we consider a different source of interaction between aggregate shocks and private allocations that arises in a dynamic environment. We assume that aggregate and private shocks are independent from one another and explore how constrained optimal insurance may still endogenously generate links in dynamic allocations. That is, we eliminate all exogenous interactions between shocks and investigate the interconnections displayed by the corresponding allocations.

Our prototype economy is a two period Mirrlees setting with a finite number of productivity levels and i.i.d. aggregate shocks. First, however, we illustrate some of the forces at work using a variation of Atkeson and Lucas (1992) economy in which aggregate risk is added to the problem. Exploiting the intuition of a two period model, we show a strong form of dependence: allocations exhibit memory with respect to aggregate shocks. That is, today's allocation depends not only on current shock, as in Wilson (1968), and on yesterday's idiosyncratic shock, as in Rogerson (1985), but also on yesterday's aggregate shock, despite it's being public and independent of idiosyncratic shocks.

Although useful for illustrating some of the underlying forces at work, it is not a priori clear that the results should extend to a dynamic Mirrlees economy. What is special about this environment is that individuals can only be separated using a dynamic incentive scheme, since in each period allocations are one-dimensional. In other words, no screening is possible in a static version of this economy, whilst it certainly is in a Mirrlees setting.

It turns out that memory of aggregate shock also arises in our two period Mirrlees economy. Our argument is more subtle than the Atkeson and Lucas (1992) case, because, as we have mentioned, incentives can be generated within each period, using the leisure/consumption trade-off.

In particular, it is worth recalling that our assumption of independence of individual productivity shocks rules out non-trivial interactions between aggregate and private risk due to the reasons explored in Holmström (1982). Instead, the rationale for our results is closer to Levin (2002). In our model, the resource constraint induces an interdependence in the provision of incentives across agents, while in his model, the same type of interdependence arises due to an endogenous restriction regarding what may be promised to the pool of agents in a credible way.²

Our results are potentially important to understand how the very nature of constrained efficient insurance for private risk may generate non-trivial dynamics for aggregate variables. It may also shed light on the macro implications of endogenously incomplete private insur-

²We thank Vinicius Carrasco for pointing this out.

ance markets. The two aforementioned models are presented in a two period setting. We strongly believe that many of the presented results are valid for any number of periods when idiosyncratic shocks are a Markov process of order one.

We illustrate our main findings with numeric exercises. We mostly explore how persistence in private shocks is negatively related to memory. From a purely idiosyncratic viewpoint, all the rationale of backloading incentives is associated with the idea that someone that had a bad shock today need not experience all the bad consequences today, if in the future she may turn out to experience a good shock. The more persistent the shock the lower the incentives for backloading. In the limit, when shocks are perfectly persistent incentives are evenly spread across periods.

With aggregate shocks, the distribution of incentives across states of nature becomes an issue as well. In our environment we define perfect insurance with respect to aggregate shock—i.e., allocations vary *with aggregate shocks* in the same fashion as in a first best world, which, in a Mirrlees world, implies perfect tax smoothing. We show this property to characterize situations with private information but no idiosyncratic risk—replicating the finding of Werning (2007)—and the last period of finite economies.³ In contrast with Werning (2007), we also consider environments with evolving private information, both in an Atkeson and Lucas (1992) and in a dynamic Mirrlees (1971) setting.

Our focus on a two period setting is restrictive and is due simply to the technical difficulties involved in extending the model to a fully dynamic environment. Indeed, although Phelan (1994) studies a fully dynamic moral hazard OLG economy subject to aggregate shocks, to keep the environment tractable, he assumes that individuals have constant absolute risk aversion—CARA—preferences and restrict private shocks to be i.i.d.. He shows that the optimal allocations depend on past aggregate variables.

We show that memory of aggregate shocks characterizes our economy, as well, and that memory is negatively related with persistence of private shocks. The fact that shocks may have persistent effects on allocations despite their public and i.i.d nature, was first shown by Phelan (1994) in a dynamic moral hazard economy with CARA preference, while the lack of memory in the extreme opposite case of perfect persistence can be hinted from the results in Werning (2007). Our numerical simulations indicate that these are not knife-edge results: there is a monotonic relationship between private persistence and aggregate memory in many different environments.

Our two period Mirrlees economy, allows us to explore other dimensions of the macro

³Also presented in the Appendix.

consequences of private information, as well. Indeed, although most of the literature of the so-called Dynamic Public Finance Literature is normative, in recent years, a relatively large body of research has arisen on the Macroeconomics implications of private information, see Sleet (2006). Noting that many different institutional arrangements may induce the constrained efficient allocation, this seems like a good departure point for relaxing complete market assumptions in a less arbitrary fashion.

We use our model to investigate two macroeconomic facts that are hard to explain with a representative agent setting. Our approach is to take the aggregate data generated by the model and see if it generates the type of 'wedges' that characterize the empirical findings regarding representative agent models.

We thus follow the lead of Chari et al. (2007) who have shown that prototype economies can be mapped into a representative agent economies with time and state varying wedges. They go on to show that the labor wedge is key to explain business cycle stylized facts.⁴ Because incentives are provided in the leisure/consumption margin in a time and state varying fashion, our prototype economy is potentially useful for understanding state-dependent labor wedges. We show that whilst our economy does generate state varying wedges, the covariation with aggregate product has the wrong sign. Optimal wedges are smaller in bad times while empirically it is the opposite.

Then we move on to investigate asset pricing. Once again, the repeated Mirrlees economy is potentially useful for accommodating asset pricing anomalies. The point here is that, although, shocks are exogenously independent, the same is not true for allocations. Moreover, agents in this economy must have restricted access to financial markets, a point that is crucial for an incomplete markets to generate any type of interesting asset pricing behavior, as emphasized by ?.

We show that the pricing kernel generated using a representative agent assumption is not too different from the true pricing kernel. As a consequence pricing errors are very small in our exercise, which is in contrast with the findings of Kocherlakota and Pistaferri (2007) or Kocherlakota and Pistaferri (2009) using real consumption and asset pricing data.

The rest of the paper is organized as follows. Section 2 describes a general setting that encompasses both economies that we investigate. The Atkeson and Lucas (1992) framework is studied in section 3 while the Mirrlees (1971) setting is dealt with in section 4. Some numerical exercises in the spirit of Golosov et al. (2006) are conducted in section 6. Section 7 concludes the paper.

⁴See also Shimer (2009).

2. Basic Setting

The economy is inhabited by a continuum of measure one of ex-ante identical individuals, each living for 2 periods. In every period agents' preferences are subject to 'taste' shocks $\theta_t \in \Theta = \{\theta(H), \theta(L)\}$ for $t = 1, 2$, with $\theta(H) > \theta(L)$. We use $\theta^2 = (\theta_1, \theta_2)$ to denote a history of idiosyncratic shocks up to period 2. Agents have preferences over a consumption set X , and rank deterministic allocation streams $\{x_t\}_{t=1}^2 \in \mathbb{R}_+^{L^2}$, given taste shocks θ^t according to

$$\sum_{t=1,2} \beta^{t-1} U(x_t, \theta_t).$$

We let $U(\cdot, \theta) : \mathbb{R}_+^L \rightarrow \mathbb{R}$ be a smooth, strictly increasing and strictly concave function, for all $\theta \in \Theta$.

Taste shocks are not the only source of uncertainty in this economy. In each period, an aggregate shock represented by a random variable $z_t \in Z \equiv \{\bar{z}, \underline{z}\}$, with $\bar{z} > \underline{z} > 0$, affects the economy's technology. Let also $z^t = (z_1, z_2)$ denote the history of aggregate shocks. We consider i.i.d. aggregate shocks that are distributed according to μ_Z^1 (μ_Z^2 is the product measure induced on Z^2).

Conditional on z^t , idiosyncratic shocks, θ^t , are drawn from the probability measure μ_Θ^2 defined over Θ^2 . We assume that a law of large numbers applies so that, at period 2 state z^t , the cross-sectional distribution of agents coincides with the ex-ante distribution μ_Θ^2 . We denote by $\mu_\Theta^1(\theta)$ the marginal distribution of first period shocks, $\sum_{\theta'} \mu_\Theta^2(\theta, \theta')$.

Idiosyncratic shocks are private information while aggregate shocks are publicly observed.

An *allocation* is, in this case, $x = \{x_t\}_{t=1}^2$, with $x_t : Z^t \times \Theta^t \rightarrow \mathbb{R}_+^L$ for each $t \in \{1, 2\}$, where $x_t(\theta^t, z^t)$ is a (θ^t, z^t) -measurable function that denotes the bundle allocated to an agent with history θ^t at period t , when aggregate history is z^t . Agents' preferences are represented by a von Neumann-Morgenstern utility function,

$$\mathbb{E} \left[\sum_{t=1,2} \beta^{t-1} U(x_t(\theta^t, z^t), \theta_t) \right] = \sum_{t=1,2} \beta^{t-1} \sum_{z^t} \sum_{\theta^t} \mu_\Theta^t(\theta^t) \mu_Z^t(z^t) U(x_t(\theta^t, z^t), \theta_t). \quad (1)$$

Technology is represented by a transformation function $G : \mathbb{R}_+^L \times Z \rightarrow \mathbb{R}$. We say that an allocation x is *resource feasible* if

$$G \left(\sum_{\theta^t} \mu_\Theta^t(\theta^t) x_t(\theta^t, z^t), z_t \right) \leq 0, \text{ for all } t, z^t. \quad (2)$$

A benevolent planner maximizes individuals' expected utility. From a period 1 perspective this is equivalent to our assuming that the government maximizes a utilitarian social

welfare function.

Due to the presence of private information we write the planner’s program as a mechanism design problem. Throughout the analysis we assume that the planner is endowed with a commitment technology. This will allow us to restrict implementation to direct revelation mechanisms, in which agents are asked to report their types at each period, and are assigned corresponding bundles.

Define a *reporting strategy*, $\sigma = \{\sigma_t\}_{t=1}^2$, as a sequence of mappings $\sigma_t : Z^t \times \Theta^t \rightarrow \Theta$, which associate to every history (z^t, θ^t) an announcement $\hat{\theta}$. Two things are worth mentioning here. First, since individuals cannot lie about the aggregate state of the economy, reports are restricted to idiosyncratic shocks. Announcement strategies may, nonetheless, depend on z^t . Second, we assume that the agent only announces the current shock, θ_2 , in the second period, and not the history θ^2 . Alternatively the mechanism could be described by requiring the agent to announce θ^2 instead. However, given that we assume perfect recall by the government, strategies that do not respect the condition $\sigma^2(\theta^2) = (\sigma^1(\theta_1), \theta')$ for some θ' are ruled out. As a consequence, asking θ_t each period is without loss of generality. The set of all admissible strategies is, then, Σ . We use $\sigma^{TT} = \{\sigma_t^{TT}\}_{t=1}^2$ to denote the truth-telling strategy $\sigma_t^{TT}(\theta^t) = \theta_t$ for all t .

We abuse notation and let $U(x, \sigma)$ be the utility derived from an agent choosing reporting strategy σ given allocation x , i.e.,

$$U(x, \sigma) \equiv \mathbb{E} \left[\sum_{t=1,2} \beta^{t-1} U(x_t(\sigma^t(\theta^t, z^t), z^t), \theta_t) \right].$$

An allocation x is *incentive compatible* if

$$U(x, \sigma^{TT}) \geq U(x, \sigma), \text{ for all } \sigma \in \Sigma. \tag{3}$$

The goal of this paper is to characterize some features of the solution to the problem of maximizing (1) subject to (2) and (3). Since the sets of all possible shocks is finite, existence and uniqueness of this solution is trivially verified for the cases of interest. From now on we denote this allocation $x^* = \{x_t^*\}_{t=1}^2$. Two relevant special cases of commodity spaces, taste shocks—Section 3—and technological shocks—Section 4—, are discussed.

3. The Preference Shock Case

We shall illustrate some of the forces underlying our result using a two period of Atkeson and Lucas (1992) with aggregate shocks. Individuals care only about consumption; $x = c \in$

\mathbb{R}_+ . We represent an allocation x as a consumption stream, denoted by $c = \{c_t\}_{t=1}^2 \in \mathbb{R}_+^2$. In each period, individuals are exposed to taste shocks that determine their marginal utility of consumption across states. That is, individuals preferences are of the form

$$U(c, \theta) = \begin{cases} \theta c^{1-\rho} / (1-\rho) & , \text{ if } \rho > 0 \text{ and } \rho \neq 1, \\ \theta \log c & , \text{ if } \rho = 1, \end{cases}$$

where θ is the taste parameter. For the moment let taste shocks be i.i.d.

Resource constraint is given by an aggregate endowment that is determined by current period's shock z_t :⁵

$$G\left(\sum_{\theta^t} \mu(\theta^t) c(\theta^t, z^t), z_t\right) = \sum_{\theta^t} \mu(\theta^t) c(\theta^t, z^t) - z_t \leq 0, \text{ for all } t, z^t.$$

As a benchmark case, consider the first best allocation,

$$c_t^{FB}(\theta^t, z^t) = z_t \theta_t^{\frac{1}{\rho}} \mathbb{E}\left[\theta^{\frac{1}{\rho}}\right]^{-1}.$$

Four important features of this allocation are noteworthy. The allocation is: *i*) independent of θ^{t-1} ; *ii*) increasing in θ_t , *iii*) independent of z^{t-1} and; *iv*) linear in z_t .

With private information, things change quite a bit. It is already known that independence of θ -history is ruled out in almost every example of repeated moral hazard or informational friction, for efficiency reasons. In particular, because temporary utility is defined in terms of a single good (consumption) in this setting, private information is revealed *only* through intertemporal trade-offs. For an allocation to be incentive compatible, more transfers today must generate lower expected transfers tomorrow.

As we have seen, a first best allocation is characterized by higher consumption for agents with higher θ . With private information, this allocation is not implementable. All agents have incentive to announce higher θ to get positive transfers. To obtain truthful revelation more consumption today must be exchanged for lower expected consumption tomorrow, which imposes history dependence on consumption. Note that this is the only available way to provide any insurance for this non-observable shock in the presented setting.

The last two features are more subtle and less explored in the moral hazard literature. Since aggregate shocks are public information and independent across time, allocation independence on previous aggregate shocks seems a natural result. This is not true, however, in a

⁵We could consider a general function $e(\cdot)$, in which case aggregate endowment in state z would be given by $e(z)$. Considering $e(z) = z$ is without loss of generality.

limited insurance environment. Optimal allocations are driven basically by the trade-off between insurance and incentives, and this is generally affected by these aggregate shocks.⁶ As a consequence, the whole distribution of future promised utility (the variable used to screen agents) depends on past aggregate shocks as they affected previous allocations. Albanesi and Sleet (2006)

Monotonicity in θ_t follows immediately from incentive compatibility since single-crossing is obviously valid here. By single-crossing, we mean that, considering only first period shocks, individuals can be ordered by the marginal rate of substitution between present and future consumption.⁷

Linearity, on the other hand, seems a rather natural characteristic to expect from an optimal allocation. Linear dependence on z_t is a direct characteristic of optimal risk sharing related to aggregate shocks in settings with the constant absolute risk aversion assumption, with each agent taking a fixed share of the risk.

When the allocation is linear in z_t , i.e., $c_t(\theta^t, z^t) = z_t \eta(\theta^t, z^{t-1})$, we say it is *separable*. Lemma 1 says that last period allocation is separable. As we shall see, this will not be the case in the first period.

Although we shall ultimately show that the planner's program may be written as a static one, thus allowing for a simple representation in terms of current and future consumption, to keep the analysis parallel to that in section XXX, it will be useful to define utility promises

$$w^*(\theta_1, z_1) \equiv (1 - \rho)^{-1} \sum_{\theta^2 > \theta_1} \sum_{z^2 > z_1} \mu_Z(z_2) \mu_\Theta(\theta) \theta c_2^*(\theta^2, z^2)^{1-\rho}.$$

In this case, truthful revelation is obtained by trading-off current consumption for utility promises.

In deriving optimal allocations, the first thing to note is that no revelation of private information takes place in the last period, since there is no further consumption to allow for screening the agents. Indeed, take any fixed $\theta', \theta'' \in \Theta$ and any $(\bar{\theta}, \bar{z}^2) \in \Theta \times Z^2$, then consider $\sigma = (\sigma_1^{TT}, \sigma_2)$ where $\sigma_2(\theta, \theta', \bar{z}^2) = \theta''$ and $\sigma = \sigma^{TT}$ otherwise, then $c_2(\bar{\theta}, \theta', \bar{z}^2) \geq c_2(\bar{\theta}, \theta'', \bar{z}^2)$. Analyzing the reverse deviation we get that $c(\bar{\theta}, \theta', \bar{z}^2) = c_2(\bar{\theta}, \theta'', \bar{z}^2)$. We, then, realize that c_2 can be represented as a function $c'_2 : \Theta \times Z^2 \rightarrow \mathbb{R}_+$. From now on we restrict the analysis to functions of this form (consumption and reporting strategies).

⁶It may also be driven by the trade-off between inequality and incentives, depending on whether the analysis is carried on before or after individual shock values are sorted across agents.

⁷See discussion in section 3.1.

Utility promises will take, in this case, the form

$$w^*(\theta_1, z_1) \equiv (1 - \rho)^{-1} \mathbb{E}(\theta) \sum_{z_2 \in Z} \mu_Z^1(z_2) c_2^*(\theta_1, z_1, z_2)^{1-\rho}.$$

Note also that, in the last period, the only important things inherited from period one are utility promises that must be fulfilled. Using c^* to denote optimal (second best) the next lemma characterize second period consumption.

Lemma 1 *Conditional on period 1 promises, consumption in period 2 presents optimal risk sharing relative to the aggregate shock, i. e., for any $(\theta_1, z_1, \theta'_1, z'_1)$, $c_2^*(\theta_1, z_1, \cdot)$ and $c_2^*(\theta'_1, z'_1, \cdot)$ are perfectly correlated. Moreover, utility promises fully define consumption distribution in period 2,*

$$c_2^*(\theta_1, z_1, z_2) = z_2 \frac{w^*(\theta_1, z_1)^{\frac{1}{1-\rho}}}{\sum_{\theta_1 \in \Theta} \mu_{\Theta}^1(\theta_1) w^*(\theta_1, z_1)^{\frac{1}{1-\rho}}}.$$

One important feature is that there is perfect aggregate risk insurance, conditional on previous utility promises. Consumption in period 2 is perfectly correlated across agents and the consumption distribution is determined, except for a constant, solely by period 1 distribution of utility promises. Given that there is no screening problem in the last period, there is no idiosyncratic risk insurance. Hence, no reason remains to distort allocations relative to aggregate shock.

The direct relationship between utility promises made in period 1 and consumption levels in period two imply that we can anticipate resource constraints in the last period by deriving the feasible subset of promises. This allows us to characterize the planner's problem as a static one.

Proposition 1 *When preference shocks are private information:*

*i) first period consumption is **not** separable on aggregate shock z_1 , i. e., there exist **no** functions $\tilde{c}(\theta_1)$ and $\eta(z_1)$ such that $c_1^*(\theta_1, z_1) = \tilde{c}(\theta_1) \eta(z_1)$, and;*

ii) second period consumption depends on period 1 aggregate shocks, i.e., $c_2 : Z^2 \times \Theta^2 \rightarrow \mathbb{R}_+$ cannot be independent of z_1 .

Proof. See Appendix. ■

The central issue is that of characterizing the trade-off between consumption inequality in the two periods. Endowment shocks in period 1 could be accommodated by increasing proportionately each agent’s consumption, which is the possibility rejected in this proposition. The reason for this is that more endowment changes the incentive versus inequality conflict in the first period.

Another interesting feature that arises from limited insurance in the presence of aggregate shocks is that allocations may present persistence regarding aggregate shocks. In other words, even though previous aggregate shocks are independent of present information structure of the economy, current consumption distribution may depend on the history of the economy. Therefore aggregate variables may present time dependence due to the presence of long term contracts, having nothing to do with primitive persistence in the model. This is proved in the next proposition.

3.1. Persistent Shocks

Although we shall only explore the case of persistent shocks in the Mirrlees setting of next section, a brief heuristic account of how the dependence on aggregate shocks may be influenced by persistence is due here.

The consequence of such observation is that the planners’ problem is effectively a static one. Some insurance will be possible provided that marginal rates of substitution can be ordered.

Let $\mathbb{E}[\theta'|\theta]$ denote the expected value of second period taste parameter θ' conditional on one’s having realized θ in period 1. The marginal rate of substitution between future and current consumption is, then,

$$\left. \frac{dc_2}{dc_1} \right|_{\bar{v}} = \frac{\theta}{\mathbb{E}[\theta'|\theta]} \frac{c_1^{-\rho}}{c_2^{-\rho}}.$$

It is apparent from the expression above that, the higher the persistence, the smaller the difference between marginal rates of substitution between individuals. In particular, when persistence is complete, i.e., $\mathbb{E}[\theta'|\theta] = \theta$, the marginal rate of substitution is identical across agents and no insurance is possible.

Because, there are no transfers at the (constrained) optimum, there is no dependence on history; in particular, on aggregate history. There is a sense in which this result is of limited interest, however. Memory with respect to aggregate shocks only disappears when individuals allocations are the ones they have in autarky. This will not be the case in the Mirrlees economy for which insurance is possible even in the last period, through the

consumption/leisure trade-off.

4. Mirrlees Economy

In this section, we consider a dynamic Mirrlees economy, where agents temporary preferences defined as a function of consumption c and effort l is of the form $u(c) - v(l)$, where $u(\cdot)$ strictly concave and increasing and $v(\cdot)$ strictly convex and increasing.

As in Mirrlees (1971), we redefine choice variables. Instead of (c, l) we consider $x = (c, y)$, where c is consumption and y denotes efficiency units of work. Now, θ is a productivity parameter which, combined with z , determines how many efficiency units an agent produces per unit of effort, l ; an agent with productivity θ produces $y = l\theta z$ with effort l if the aggregate state is z . Because our commodity space is now defined in terms of x , an agent θ can be viewed as a taste shock, just as in the general presentation of the problem in Section 2. The technology is once again linear, one efficiency unit of labour produces one unit of consumption.⁸

The social Planner's problem is

$$\max \sum_{t=1,2} \beta^{t-1} \sum_{\theta^t} \sum_{z^t} \mu_z^t(z^t) \mu_{\Theta}^t(\theta^t) \left[u(c(\theta^t, z^t)) - v\left(\frac{y(\theta^t, z^t)}{\theta_t z_t}\right) \right]$$

subject to

$$\sum_{\theta^t} \mu_{\Theta}^t(\theta^t) [c(\theta^t, z^t) - y(\theta^t, z^t)] \leq 0$$

and

$$\begin{aligned} & \sum_{t=1,2} \beta^{t-1} \sum_{\theta^t} \sum_{z^t} \mu_z^t(z^t) \mu_{\Theta}^t(\theta^t) \left[u(c(\theta^t, z^t)) - v\left(\frac{y(\theta^t, z^t)}{\theta_t z_t}\right) \right] \geq \\ & \sum_{t=1,2} \beta^{t-1} \sum_{\theta^t} \sum_{z^t} \mu_z^t(z^t) \mu_{\Theta}^t(\theta^t) \left[u(c(\sigma(\theta^t, z^t), z^t)) - v\left(\frac{y(\sigma(\theta^t, z^t), z^t)}{\theta_t z_t}\right) \right] \end{aligned}$$

for all $\sigma \in \Sigma$.

⁸To map technology exactly in the terms of Section 2 we would instead define $\tilde{y} = l\theta$ and let

$$G\left(\sum \mu_{\Theta}(\theta)c(\theta, z), \sum \mu_{\Theta}(\theta)\tilde{y}(\theta, z), z\right) = \sum \mu_{\Theta}(\theta)c(\theta, z) - \sum \mu_{\Theta}(\theta)\tilde{y}(\theta, z)z \leq 0.$$

As it turns, the definition of variables we adopted is more convenient.

We shall now investigate how the optimal allocation (c^*, l^*) depends on the history of aggregate shocks z^t for each t . More specifically, whether aggregate shocks generate persistent effects on the distribution of consumption and labor. In other words we ask: what is the prescription for optimal tax response to aggregate shocks?

It is a general feature of repeated screening and moral hazard problems that allocations keep track of individuals' private shocks histories. This allows the principal to go beyond repeating the static optimal allocation by linking future allocations to present type reports.⁹

This whole argument is false for the aggregate shock for one simple reason: it is observable. Since the aggregate state of the economy is assumed to be known by everyone, and is unrelated with idiosyncratic shocks, there is no obvious reason to believe that the optimal allocation will present history dependence with respect to this variable.

The main finding of this section is that aggregate shocks may present persistence, even in this simple and, in many senses, “independent” setting. We show that, for our simple example, this dependence is indeed optimal.

At any given period, individuals care about the current allocation and how the current report will affect their prospects in further periods. Lemma 2, below, states that, since it is always possible to distort future consumption to separate individuals today, differences in current consumption are preserved in the future in such a way as not to interfere with future incentive compatibilities.

Proposition 2 *The optimal allocation satisfies*

$$\frac{u'(\bar{\theta}^t, z^t)}{u'(\theta^t, z^t)} = \frac{\mathbb{E} [u'(c_{t+1})^{-1} | \bar{\theta}^t, z^{t+1}]^{-1}}{\mathbb{E} [u'(c_{t+1})^{-1} | \theta^t, z^{t+1}]^{-1}}, \quad (4)$$

for every z^t and $\bar{\theta}^t, \theta^t$.

Proof. From Proposition 1 in Kocherlakota (2005), we know that there exists $\{\lambda_t\}_{t=1}^T$, with

⁹The extreme poverty result presented in Atkeson and Lucas (1992) and Phelan (1998) is a long run consequence of this feature of optimal allocations. At each period, the cross-sectional distribution of consumption and labor inherits the heterogeneity of previous period and brings a new source of variation, since you have to generate further distortions to separate agents with different types at the current period. The extra dispersion in allocations is independent of previous heterogeneity, assuming i.i.d. individual shocks, an assumption present in most of the literature. This pattern generates ever increasing inequality across agents.

$\lambda_t : Z^t \rightarrow \mathbb{R}$, such that

$$\lambda_{t+1}(z^{t+1}) = \frac{\mathbb{E} [u'(c_{t+1})^{-1} | \theta^t, z^{t+1}]^{-1}}{u'(c_t(\theta^t, z^t))},$$

for all z^{t+1} and θ^t . Dividing this equation for $\bar{\theta}^t$ and θ^t implies the Lemma. ■

There are a couple of things worth noting about this result. First, the expectations in both the numerator and denominator in the left hand side of (4) are conditional on z^{t+1} . That is, for every continuation, z^{t+1} , of current history, z^t , the harmonic mean of future marginal utility of consumption is equal to the current marginal utility of consumption for all agents.

Second, this result is valid in a much more general setting, without any restrictions on the number of periods, in particular, infinite periods are allowed, or the nature of the stochastic process, which need not be Markov, for example.

One interesting and straightforward example of this result is the case in which $u(\cdot)$ is log.

Corollary 1 *Consider $u = \ln$, then, letting $Y(z^t)$ denote total production, we have*

$$\frac{c_t(\bar{\theta}^t, z^t)}{Y(z^t)} = \frac{\sum_{\theta^t} \mu(\theta^t | \bar{\theta}^t) c_{t+1}((\bar{\theta}^t, \theta^t), (z^t, z))}{\sum_{\theta^{t+1}} \mu(\theta^{t+1}) c_{t+1}(\theta^{t+1}, (z^t, z))} = \frac{\mathbb{E}[c_{t+1} | \bar{\theta}^t, z^{t+1}]}{Y(z^{t+1})}.$$

The fact that the expectation in the right hand side of the expression above is conditioned on z^{t+1} allows for a simple interpretation of the expression above. Namely, the expected share of aggregate output an agent gets to consume in any aggregate state tomorrow is equal to the share of current output that she consumes today. This means that, two agents are entitled to different shares of total output today, they will necessarily receive different shares of total output for at least one realization of private shocks and this will be true for all aggregate states.

More importantly for our discussion is the fact that if, the share an agent with private history θ^t is entitled to in aggregate state z_t differs from the share she would be entitled to in case state \hat{z}_t realized, instead, then the expected share she will get in (z^{t-1}, z_t, z) will differ from the expected share she gets in (z^{t-1}, \hat{z}_t, z) .

The consequence of this corollary is that, with logarithmic preferences, it is either the case that optimal shares are independent of z or they must display memory with regards

to aggregate shocks. We shall explore these possibilities considering power utility, of which log is a special case. That is, we restrict analysis to specific utility functions with constant absolute risk aversion in consumption and power disutility of labor effort,

$$u(c) = \frac{c^{1-\rho}}{1-\rho} \text{ and } v(l) = \frac{l^\gamma}{\gamma}, \quad (5)$$

for $\rho > 0$, $\rho \neq 1$ and $\gamma > 1$. For $\rho = 1$ take $u(c) = \log c$.

Given the low tractability of incentive compatibility constraints in even simple environments, it has become relatively common to restrict analysis substantially in terms of preferences and stochastic distribution of shocks in order to obtain clear-cut results. We believe this is an interesting case, because it is a common preference structure in the Macroeconomic literature and, on both the static Mirrlees and representative agent model.

In the case of a representative agent with preferences as in (5) and no savings technology, we have $C(z) = Y(z) = z^{\gamma/(\gamma+\rho-1)}$. As we shall show later, in a repeated Mirrlees model, along the lines of Werning (2007), constrained efficient allocations can be written $c(\theta, z^t) = \tilde{c}_t(\theta) \eta(z_t)$ and $y(\theta, z^t) = y(\theta) \eta(z_t)$, where $\eta(z_t) \equiv z_t^{\gamma/(\rho+\gamma-1)}$. This means that idiosyncratic and aggregate states have separable effects in individual consumption and labor allocations. It is also an interesting point to compare how the dynamic structure of private information and the optimal social insurance scheme will affect this behavior.

We now show that it is optimal to generate dependence on previous aggregate shocks as an incentive generation mechanism.

Proposition 3 *Consider a finite Mirrlees economy with $T = 2$ periods and i.i.d. z -shocks.*

Then,

i) if $\mu_\Theta^2(\theta^2) > 0$ for all θ^2 , period 2 allocations depend on z_1 , and;

ii) there exist functions \tilde{c} , \tilde{y} such that

$$c_T^*(\theta^T, z^T) = \tilde{c}_T(\theta^T, z^{T-1}) z_T^{\frac{\gamma}{\gamma+\rho-1}},$$

and

$$y_T^*(\theta^T, z^T) = \tilde{y}_T(\theta^T, z^{T-1}) z_T^{\frac{\gamma}{\gamma+\rho-1}}.$$

Proof. See Appendix. ■

Since in the last period there is no future allocations to be also used to screen agents, allocations depend on aggregate shocks as in a static Mirrlees model. We say that the last

period allocation is *separable* in this case. An important implication of this separability of individual and public risk is that marginal distortions, which mean marginal labor tax, are independent of shocks.

Corollary 2 *Period T marginal labor tax does not depend on z_T .*

Proof. From the definition of marginal tax

$$\tau_T^* (\theta^t, z^t) = 1 - \frac{y_T^* (\theta^T, z^T)^{\gamma-1}}{(\theta_T z_T) c_T^* (\theta^T, z^T)^{-\rho}},$$

it follows immediately that

$$\begin{aligned} \tau_t^* (\theta^t, z^t) &= 1 - \frac{z_T^{\frac{\gamma(\gamma-1)}{\gamma+\rho-1}} \tilde{y}_t (\theta^T, z^{T-1})^{\gamma-1}}{(\theta_T z_T)^\gamma z_T^{\frac{-\rho\gamma}{\gamma+\rho-1}} \tilde{c}_T (\theta^T, z^{T-1})^{-\rho}} \\ &= 1 - \frac{\tilde{y}_t (\theta^T, z^{T-1})^{\gamma-1}}{\theta_T^\gamma \tilde{c}_T (\theta^T, z^{T-1})^{-\rho}}, \end{aligned}$$

which is independent of z_T . ■

4.1. Perfectly Persistent Shocks

Now consider the case in which $\mu_\Theta^2 (i, i) = 1$, $i = H, L$, i.e., individual productivity levels are constant. Then, the first order conditions of the Planner's problem are given by

$$\begin{aligned} \left[1 + \sum_{\theta'} [\phi (\theta'|\theta) - \phi (\theta|\theta')] \right] c_t (\theta, z^t)^{-\rho} &= \lambda (z^t), \\ \left[1 + \sum_{\theta'} \left[\phi (\theta'|\theta) - \phi (\theta|\theta') \left(\frac{\theta}{\theta'} \right)^\gamma \right] \right] \frac{y_t (\theta, z^t)^{\gamma-1}}{(\theta z_t)^\gamma} &= \lambda (z^t), \end{aligned}$$

where ϕ are the IC constraints multipliers.

Then we have that for arbitrary κ and ω , $c_t (\theta, z^t) = \lambda (z^t)^{\frac{-1}{\rho}} \kappa (\theta)$ and $y_t (\theta, z^t) = (\theta z_t)^{\frac{\gamma}{\gamma-1}} \lambda (z^t)^{\frac{1}{\gamma-1}} \omega (\theta)$. Substituting this in the resource constraints, that always binds, we have that

$$\sum \mu (\theta) \left[\lambda (z^t)^{\frac{-1}{\rho}} \kappa (\theta) - (\theta z_t)^{\frac{\gamma}{\gamma-1}} \lambda (z^t)^{\frac{1}{\gamma-1}} \omega (\theta) \right] = 0,$$

which means that $\lambda(z^t) = a z_t^{-\frac{\gamma\rho}{\rho+\gamma-1}}$, and then,

$$c_t(\theta, z^t) = \tilde{c}(\theta) z_t^{\frac{\gamma}{\rho+\gamma-1}}, \quad y_t(\theta, z^t) = \tilde{y}(\theta) z_t^{\frac{\gamma}{\rho+\gamma-1}}.$$

Suppose that the choice of variables for consumption and labor are \tilde{c} and \tilde{y} , while the *de facto* allocation is given by $x_t(\theta^t, z^t) = z_t^{\frac{\gamma}{\rho+\gamma-1}} \tilde{x}_t(\theta^t)$. Since this does not affect the degree of freedom in the planner's problem, it is basically the same choice.

Therefore, the problem can be rewritten as

$$\max \mathbb{E} \left[z_t^{\frac{\gamma(1-\rho)}{\rho+\gamma-1}} \right] \sum_{t=1,2} \beta \sum_{\theta} \mu_{\Theta}^1(\theta) \left[\frac{\tilde{c}(\theta)^{1-\rho}}{1-\rho} - \frac{\tilde{y}(\theta)^\gamma}{\gamma\theta^\gamma} \right]$$

subject to

$$z_t^{\frac{\gamma}{\rho+\gamma-1}} \sum_{\theta^t} \mu_{\Theta}^1(\theta) [\tilde{c}(\theta) - \tilde{y}(\theta)] \leq 0$$

and

$$\begin{aligned} & \mathbb{E} \left[z_t^{\frac{\gamma(1-\rho)}{\rho+\gamma-1}} \right] \sum_{t=1,2} \beta^{t-1} \sum_{\theta^t} \mu_{\Theta}^1(\theta) \left[\frac{\tilde{c}(\theta)^{1-\rho}}{1-\rho} - \frac{\tilde{y}(\theta)^\gamma}{\gamma\theta^\gamma} \right] \geq \\ & \mathbb{E} \left[z_t^{\frac{\gamma(1-\rho)}{\rho+\gamma-1}} \right] \sum_{t=1,2} \beta^{t-1} \sum_{\theta^t} \mu_{\Theta}^1(\theta) \left[\frac{\tilde{c}(\theta')^{1-\rho}}{1-\rho} - \frac{\tilde{y}(\theta')^\gamma}{\gamma\theta'^\gamma} \right]. \end{aligned}$$

It is not hard to see that Pareto optimal allocations are given by

$$x_t^*(\theta, z^t) = x^M(\theta) z_t^{\frac{\gamma}{\rho+\gamma-1}},$$

where x^M is the optimal allocation in the static Mirrlees problem in the analogous economy with $z = 1$. This result does not depend on the number of periods, however it is very sensitive to the assumption about preferences. In the case of non isoelastic disutility of labor, for example, it is not valid anymore.

From this analysis, two immediate results follow.

Proposition 4 *If $\mu_{\Theta}(i, j|i) = 0$ whenever $i \neq j$, then period 2 allocations do not depend on z_1 .*

Define the implicit optimal marginal labor tax as

$$\tau_t^*(\theta^t, z^t) = 1 - \frac{y_t^*(\theta^t, z^t)^{\gamma-1}}{(\theta_t z_t)^\gamma c_t^*(\theta^t, z^t)^{-\rho}}.$$

And it is simple to conclude that optimal tax schedule presents perfect tax smoothing in every period.

Corollary 3 *If $\mu_{\Theta}(i, j|i) = 0$ whenever $i \neq j$, then*

$$\tau_t^*(\theta^t, z^t) = 1 - \frac{y^M(\theta^T, z^{T-1})^{\gamma-1}}{\theta_t^\gamma c^M(\theta^T, z^{T-1})^{-\rho}}$$

These results suggest a negative relationship between individual shock persistence and aggregate allocation persistence, at least for the extreme case of i.i.d and perfectly persistent shocks. Our objective in the next section is to show that these are not knife-edge cases. We provide a numerical example in which aggregate memory declines continuously with persistence in idiosyncratic shocks.

5. Macro Consequences of Private Memory

Although most of the New Dynamic Public Finance Literature has been of a normative nature, a few attempts have been made to describe and test the macroeconomic consequences of some results, e.g., Kocherlakota and Pistaferri (2007), Kocherlakota and Pistaferri (2008), Kocherlakota and Pistaferri (2009), Ales and Maciero (2008) and Sleet (2006). The underlying idea seems to be that, because many different institutions can implement optimal allocations, it is a good departure point to imagine that whatever institutions are there, they are such that observed allocations are (constrained) optimal.

In this section we follow this strand of the literature and ask the following question. From a micro perspective we would want to follow agents through time and check whether the properties of individual allocations adhere to the data. We focus instead on the macro consequence of our model.

In a recent work, Chari et al. (2007) have shown how equilibrium allocations for disaggregated model economies are equilibrium allocations for a representative agent economy with suitably defined time-varying wedges. They go on to identify which particular wedges are more important to generate the observed behavior of macroeconomic aggregates in the representative agent models.

The current model is particularly useful in this regard for it endogenously generates wedges at an individual level. Whether these individual wedges imply a time-varying labor wedge for the representative agent, and whether this variation follows the pattern found in

the data is one of the questions we try to answer in our numeric exercises. Before moving on to those results, some definitions and preliminary results are, however, needed.

We start by defining the aggregate variables we are going to use. Aggregate consumption is $C(z^t) \equiv \sum \mu_{\Theta}(\theta^t)c(\theta^t, z^t)$, while output is $Y(z^t) \equiv \sum \mu_{\Theta}(\theta^t)y(\theta^t, z^t)$. Because there is no aggregate savings in our economy, $C(z^t) = Y(z^t)$ for all z^t .

5.1. Asset Pricing

Kocherlakota and Pistaferri (2007) and Kocherlakota and Pistaferri (2009) have shown how the pricing kernel associated with a dynamic Mirrlees economy is compatible with the behavior of excess returns in foreign and domestic markets. Using the fact that the pricing kernel is a function of the cross-sectional harmonic mean of marginal consumption—as derived by Kocherlakota (2005)—they use cross-sectional consumption data to estimate the preference parameters associated with a CRRA specification for three economies. Kocherlakota and Pistaferri (2009) show that a low coefficient of relative risk aversion is needed to account for the equity premium found in the data, while Kocherlakota and Pistaferri (2007) find similar low coefficients can account for the forward exchange rate premium.

A natural next step in this agenda is to calibrate such models to generate pricing data compatible with the underlying primitives of the economy.¹⁰ It, thus, becomes important to understand how private information regarding idiosyncratic shocks interact with public aggregate ones to generate optimal allocations from which the pricing kernels are derived. This paper considers very simple two period economies that display these elements to explore how the dynamic nature of incentives induces persistence in allocations with respect to aggregate shocks. Hence, we are not doing a true calibration exercise, but pointing out the potential results to expect from such exercises.

First, to focus on asset pricing, we may define the 'representative agent's pricing kernel', $\Lambda(z^{t+1}) = C(z^{t+1})^{-\rho}/C(z^t)^{-\rho}$. Following the idea of using parameters estimated from micro-data to 'calibrate' the model, we may compare Λ with the 'true' pricing kernel,¹¹

$$Q(z^t|z^{t-1}) = \left[\sum_{\theta^t \succ \theta^{t-1}} \mu_{\Theta}(\theta^t|\theta^{t-1}) \frac{c(\theta^t, z^t)^{\rho}}{c(\theta^{t-1}, z^{t-1})^{\rho}} \right]^{-1}.$$

¹⁰Grossly speaking, while their exercises are akin to Hansen and Singleton (1982), the calibration exercise would be akin to Mehra and Prescott (1985).

¹¹Alternatively we may search for the parameter ρ that best matches the data. In our exercises, we found $\hat{\rho} = \text{?????}$

As we have shown, last period allocation exhibits a form of separability with respect to last period shock that has interesting consequences for the form the kernel varies with last period’s shock. Another interesting feature has to do with the fact that heterogeneity per se has consequences for the magnitude of the pricing kernel, but not with the way in which it varies with the states of nature.

Using the pricing kernel $Q(z^t|z^{t-1})$ we can also derive the return of a risk free assets and the expected return of stocks.¹²

5.2. The Labor Wedge

The second issue we shall address is the labor wedge. Chari et al. (2007) show the important role played by the so-called labor wedge in generating the observed pattern of ‘choices’ for the representative agent through the business cycles. Many potential explanations for these wedges are considered in Shimer (2009) . We ask whether the underlying informational problems that characterize a repeated Mirrlees economy should be seriously considered as a potential explanation.

Chari et al. (2007) and Shimer (2009) point out to the fact that the labor wedge increases in economic downturns and decreases in good times. A dynamic Mirrlees economy may potentially allow one to take the real business cycle literature idea that fluctuations are optimal responses to real technological shock one step further and ask whether labor wedges are optimal (constrained efficient, in this case) responses to real shocks in a world where asymmetric information constrains the possibility of full insurance.

Note that one strand of the literature assumes that tax shocks may ultimately cause business cycle, thus the observed correlation between recessions and labor wedge. In the current model, instead, state varying wedges (are they to exist) are optimal responses to real shocks in a second best world.

To define the economy’s productivity we need first to define aggregate hours,

$$L(z^{t-1}, z) \equiv \sum_{\theta^{t-1}} \sum_{\theta} \mu_{\Theta}(\theta^{t-1}, \theta) \frac{y(\theta^{t-1}, \theta, z^{t-1}, z)}{\theta z}$$

and, finally, $W(z^t) = Y(z^t)/L(z^t)$.

¹²We take the economy’s GDP to represent the stock’s dividends. This idea has been recently criticized by, among others, advocates of long run risks as the relevant cause of most problems with the Consumption Capital Asset Pricing Model.

We shall endow the representative agent with the same preferences as individuals. Hence,¹³

$$\mathcal{W}(C(z), Y(z), W(z)) = \frac{C(z)^{1-\rho}}{1-\rho} - \frac{Y(z)^\gamma}{\gamma W(z)^\gamma}.$$

Note also that without taxes, the representative agent's optimal labor/consumption choice would be given by $W(z)^\gamma C(z)^{-\rho} = Y(z)^{\gamma-1}$. Naturally, there are taxes, and this equality should not be observed in practice. We shall, then, define the labor wedge,

$$\tau(z) = 1 - W(z)^{-\gamma} Y(z)^{\rho+\gamma-1},$$

where we have used the fact that $C(z) = Y(z)$.

5.3. Perfectly Persistent Shocks

Before moving on to the numeric exercises, we consider the case of perfectly persistent shocks. This is the case where heterogeneity but not idiosyncratic risks play a role. Our first result is that, in a repeated static Mirrlees economy, which we have been calling the perfect persistence case, the wedge is constant across states of nature.

We have seen that optimal allocations are of the form $c(\theta, z^t) = \tilde{c}(\theta)\eta(z_t)$, and $y(\theta, z^t) = \tilde{y}(\theta)\eta(z_t)$. In this case, $Y(z^t) = \eta(z_t) \sum \mu_\Theta(\theta)\tilde{y}(\theta)$. Hours, are, then

$$L(z^t) = \frac{\eta(z_t)}{z_t} \sum \mu_\Theta(\theta) \frac{\tilde{y}(\theta)}{\theta},$$

while the estimated productivity is

$$W(z^t) = \frac{Y(z^t)}{L(z^t)} = z_t \frac{\sum \mu_\Theta(\theta)\tilde{y}(\theta)}{\sum \mu_\Theta(\theta)\tilde{y}(\theta)/\theta}.$$

Deriving the wedge is now a simple task

$$\tau(z) = 1 - \left\{ \frac{\sum \mu_\Theta(\theta)\tilde{y}(\theta)/\theta}{\sum \mu_\Theta(\theta)\tilde{y}(\theta)} \right\}^\gamma \left\{ \sum \mu_\Theta(\theta)\tilde{y}(\theta) \right\}^{\rho+\gamma-1},$$

where we used the fact that $\eta(z_t)^{\rho+\gamma-1} z_t^\gamma = 1$.

¹³In the spirit of a calibration exercise in which instead of allowing for free parameters, we take them from micro data. See Chari et al. (2009)

As is clear from the right hand side of the expression above, the labor wedge is invariant with respect to z . We had already shown that labor wedges for each individual do not vary with z . This does not imply that the aggregate wedge will not vary. Changes in the distribution of income may generate such variation through composition effects.

The message we get from this case is that simply allowing for heterogeneity will not do. We need partially insured private risks to generate variation in wedges. In the next section we numerically explore how wedges vary with z .

A similar result obtains for asset pricing. For brevity we omit the discussion of this case.

6. Numeric Exercises

In this section we conduct numerical exercises that help illustrate some of the properties that we have derived in section 2. This exercise is similar to the one presented in Golosov et al. (2006), in which a two period Mirrlees economy with government spending shocks, that are public information. However, they have a different focus and only characterize how last period shocks affect optimal labor tax wedges. More precisely, Golosov et al. (2006) do not allow public shocks in the first period, which would allow them to address the persistence issues that is the focus of our work.

Kocherlakota (2005) provides a similar numerical example in many ways, but analyses the impact of individual shock persistence and the size of the public shocks on capital taxation, which is the main focus of his paper, and does not mention labor tax nor the possibility of persistent effects of aggregate shocks.

In our exercises we fix $1/\rho = \gamma = 2$, $\theta(L) = 1$, $\theta(H) = 2$, $\mu_{\Theta}^1(L) = \mu_{\Theta}^1(H) = .5$, $\mu_{\underline{Z}}^1(\underline{z}) = \mu_{\underline{Z}}^1(\bar{z}) = .5$, $\underline{z} = 1$ and $\bar{z} = 2$. Our measure of persistence, $\pi \in [0, 1]$, is the value of the non-unit eigenvalue of the transition matrix for idiosyncratic shocks.¹⁴ The probability of maintaining the same productivity in the next period, i. e., $\mu_{\Theta}(\theta, \theta|\theta) = \pi$ for all θ is directly associated with our measure in this two type context. In particular, because we vary π in $[0.5, 1]$, our persistence measure ranges between 0 and 1: 0 in the i.i.d. case and 1 in the case of perfectly persistent shocks.

We analyze how relevant individual shock persistence is on the allocation and what

¹⁴We use the term transition matrix in the traditional sense for Markov chains. In our case the Markov nature of idiosyncratic shock is trivial.

is driving the non-existence of persistence of aggregate shocks effects in the model for the perfectly persistence case.

In the first set of figures¹⁵ describes first period allocations. Not unexpectedly, consumption is increasing in aggregate productivity for all agents. Also, in both states agents who realize a high type have more consumption than agents who realize a low type. Low productivity individuals face a positive marginal tax rate, while high productivity individuals face a 0 marginal tax rate. This reproduces the classic result of a static Mirrlees economy. Our emphasis in those figures is on the way allocations vary with persistence. It is apparent that while consumption for low productivity individuals decrease with persistence, it increases with persistence for a high productivity agent. The more persistent a shock is, the less one gains from backloading incentives. For the exact same reason, income displays the opposite behavior, although it varies with persistence much less significantly than income. A consequence of this behavior for consumption and income is that temporary utility declines with persistence for low productivity individuals and increases for high productivity individuals. It happens so dramatically that for very low levels of persistence the temporary utility is higher for the low productivity individuals.¹⁶ Of course, for this type of utility reversion to take place in an incentive compatible way, promised utilities behave in the exact opposite way.

The second set of figures describes consumption patterns and marginal tax rates in the second period. The first thing to note is that consumptions is higher for either type if they realized a higher productivity in the first period. This is the very essence of smoothing incentives. An agent, who announced to be a high type, traded-off consumption in the first period for more consumption in the second period, for all possible idiosyncratic and aggregate realizations. This effect reduces with persistence and vanishes when persistence is full. Marginal tax rates are, as predicted, only positive for those who realize a low type in the second period. They are declining if the agent realized a low type in the first period, and are increasing if the agent realized a high type. The rationale is that, as the probability of one lying increases (since it is more and more unlikely that someone who had high productivity has now low productivity) higher distortions are imposed.¹⁷

¹⁵In the plotted charts, if X is a period one variable, $X1H$ stands for the value for the low type (represented by 1) in the high aggregate state (represented by H). For period two variables, if we take for example, $X12HL$, it represents the variable for an agent with idiosyncratic history (L, H) in the aggregate state (\bar{z}, \underline{z}) . Analogous meanings apply to all $XijIJ$, for $i, j = 1, 2$; $I, J = \bar{z}, \underline{z}$.

¹⁶To make it sound a little less paradoxical, recall that the utility for a low type is higher in the first best.

¹⁷These high marginal taxes apply to a vanishing fraction of population.

Allocations displays memory of previous aggregate shocks, in our simple model, in the sense that second period consumption depends on first period aggregate shock. Better aggregate states in the past are associated with smaller differences in consumption of low and high type, which points out to the fact that the value of backloading incentives in the first period is higher in bad economic states. In any case, memory does not seem to be quantitatively relevant.

The way consumption varies across states and types is shown in a more dramatic form in the third set of figures. We compare, in percentage terms, consumption in each possible second period state with consumption in the first period. Consumption growth declines with persistence for those who realized high productivity in the first period and increases with persistence for those who realized a low type.

The next two sets of figures deal with the macro consequences of private memory. We start with labor wedge. The first thing one notice from the fourth set of figures is that, despite all the action at a micro level, aggregate variables seem to vary only due to variations in z . This is not to say that we can go from, for example $Y(\bar{z})$ to $Y(\underline{z})$ by multiplying $Y(\bar{z})$ by \underline{z}/\bar{z} since hours respond to aggregate productivity. The point is simply that, at a first sight, the model does not seem to generate predictions that differ starkly from what one would get from a representative agent. Note however, that one must be careful when aggregating since aggregate productivity is always higher than $\mathbb{E}[\theta]$, reflecting the fact that higher productivity agents work more.

As for labor wedges, they do vary with output in the first period, provided that persistence is not full. As we have shown they do not vary with current (although they do vary with previous) output in the second period. The trouble from an empirical standpoint is that they vary in the opposite direction from what has been documented; see Shimer (2009). Many of the results we have reported in this figure may be simply due to $\rho < 1$, so it may still be possible to generate countercyclical labor wedges. What is clear from our results is that heterogeneity per se does not help.¹⁸ We need idiosyncratic risk.

Finally, with regards to asset pricing, the representative agent mis-prices both, the risk free bond and the stock if persistence is not full. The amount of mis-pricing does not seem to be very important when compared to the absolute size of the returns we are considering. Once again, any difference in the behavior of a representative consumer when compared to our model depends on the existence of idiosyncratic risk. Finally note that the model does generate state varying risk premia, but very similar to what we would observe with a

¹⁸We must, however, emphasize that aggregation is very misleading when it comes to wedges and taxes even in the pure heterogeneity case.

representative agent.

7. Conclusion

We have studied economies with both aggregate and privately observed idiosyncratic risks. We have shown that constrained efficient allocations depend on aggregate shocks in a non-trivial fashion. Allocations display memory with respect to *aggregate* shocks although we have imposed independence across periods and from idiosyncratic shocks.

The two models presented herein are very simple and do not present a full dynamic model with many periods. There is still much to be done to advance our comprehension of the intricate relationship between these two forms of insurance and their macroeconomic implications. Indeed, it is our belief that the properties emphasized here applies to more general settings and are of great interest and applicability to general macroeconomic models with any kind of aggregate stochastic shocks and some source of limited insurance related to unobservable characteristics.

As of this moment, not much research on the interrelation between idiosyncratic and public shocks at this level. As far as we know a similar exploration of any model with only consumption, as presented in section 3, has been considered by Phelan (1994), in a context with CARA preferences, and more recently by XXXX. Phelan (1994) focused on showing how some simplifying assumptions (CRRA preferences are crucial here) allows for the use of recursive methods to a setting with both idiosyncratic and aggregate shocks. His analysis does provide results regarding the interaction between the two shocks in the i.i.d. case, for his environment although the result is not emphasized.

The Mirrlees economy presented in section 4 is a special case of the model studied in Kocherlakota (2005). We have made more restrictive assumptions since the analysis of the effects of aggregate shocks and its relations to limited insurance depend on the nature of the shocks and its stochastic process. We must, however, mention the numerical exercises conducted by Golosov et al. (2006) in a two period Mirrlees economy with aggregate risk. Some of our results may be viewed as providing theoretical grounds for their numerical findings. Golosov et al. (2006) do not exploit models that generate memory of aggregate shocks. Also related are the numerical exercises found in Kocherlakota (2005). Since his concern is tax implementation, he does not focus on the effects of persistence.

We have tried to motivate further study of dynamic settings with mixed shocks because we still do not have clear conclusions about how optimal allocations look like in general. If we are to advance serious tests about, say, the existence of institutional networks that

provide optimal allocation of risk with micro level data, as in Townsend (1994), a better understanding of how observable implications are dependent or sensitive to this assumptions is needed.

As for the macro consequences. On the one hand, our results show that the covariance of the labor wedge with respect to aggregate output has the wrong sign. Asset pricing implications of our economy also do not seem to diverge too much from the aggregate consumer's one. This result, which is in contrast with Kocherlakota and Pistaferri (2007), Kocherlakota and Pistaferri (2008) and Kocherlakota and Pistaferri (2009), should be taken with some caution. First, in our model the distribution of productivity is independent of aggregate shock, which does not seem to fit the data. Second, we take dividends to be a claim to GDP, a procedure that has been under increasing criticism. Finally we have assumed a much lower degree of risk aversion.

Ours is a first step toward understanding the potential gains from allowing this type of endogenous market incompleteness in understanding macroeconomic patterns. Our preliminary findings is that the type of frictions that are assumed in a Mirrlees economy do not suffice to generate empirically plausible business cycle wedges.

A. Appendix: The Atkeson-Lucas Economy

Lemma 2 *First period optimal allocations (c_1^*, w^*) solve the following problem*

$$\max_{w, c_1} \sum_{z_1 \in Z} \mu_Z^1(z_1) \left[\sum_{\theta_1 \in \Theta} \mu_{\Theta}^1(\theta_1) \left[\theta_1 \frac{c_1(z_1, \theta_1)^{1-\rho}}{1-\rho} + \beta w(z_1, \theta_1) \right] \right] \quad (\text{A1})$$

s.t.

$$\sum_{\theta \in \Theta} \mu_{\Theta}^1(\theta) c_1(\theta, z_1) \leq z_1, \forall z_1 \in Z, \quad (\text{A2})$$

$$\theta_1 \frac{c_1(\theta_1, z_1)^{1-\rho}}{1-\rho} + \beta w(z_1, \theta_1) \geq \theta_1' \frac{c_1(\theta_1', z_1)^{1-\rho}}{1-\rho} + \beta w(z_1, \theta_1'), \forall (\theta_1, \theta_1', z_1) \in \Theta^2 \times Z, \quad (\text{A3})$$

and

$$\sum_{\theta_1 \in \Theta} \mu_{\Theta}^1(\theta_1) w(\theta_1, z_1)^{\frac{1}{1-\rho}} = \kappa. \quad (\text{A4})$$

Proof. The direct relationship between utility promises made in period 1 and consumption levels in period two imply that we can anticipate resource constraints in the last period by deriving the feasible subset of promises. I.e.,

$$\sum \mu_Z^1(z_2) \frac{1}{1-\rho} \left[\frac{z_2 w(\theta_1, z_1)^{\frac{1}{1-\rho}}}{\sum_{\theta_1 \in \Theta} \mu_\Theta^1(\theta_1) w^*(\theta_1, z_1)^{\frac{1}{1-\rho}}} \right]^{1-\rho} = w(\theta_1, z_1) \text{ for all } \theta_1, z_1.$$

This is equivalent to

$$\sum_{\theta_1 \in \Theta} \mu_\Theta^1(\theta_1) w(\theta_1, z_1)^{\frac{1}{1-\rho}} = \left[\sum \mu_Z^1(z_2) \left(\frac{1}{1-\rho} \right) z_2^{1-\rho} \right]^{\frac{1}{1-\rho}} \equiv \kappa.$$

Since any distribution of promises that satisfy this equation can be fulfilled in the following period, the problem can be stated as the one presented in the statement of the proposition for which consumption and promises choices. ■

Proof. [of Lemma 1] Given promises (w^*, c_2^*) is *sequentially efficient* in the following sense: there is no other consumption c_2' such that

$$w^*(\theta_1, z_1) = \mathbb{E}(\theta) \sum_{z_2 \in Z} \mu_Z^1(z_2) \frac{c_2'(\theta_1, z_1, z_2)^{1-\rho}}{1-\rho}; \sum_{\theta_1 \in \Theta} \mu_\Theta^1(\theta_1) c_2'(\theta_1, z_1, z_2) \leq z_2, \text{ for all } z^2 \in Z^2;$$

with the resource constraint slack for at least one $z \in Z$.

Then, there are nonnegative state prices $\lambda(z^2)$ such that

$$c_2^* \in \arg \min_{c_2} \sum_{z^2 \in Z^2} \lambda(z^2) \sum_{\theta_1 \in \Theta} \mu_\Theta^1(\theta_1) c_2(\theta_1, z_1, z_2)$$

subject to

$$\mathbb{E}(\theta) \sum_{z_2 \in Z} \mu_Z^1(z_2) \frac{c_2(\theta_1, z_1, z_2)^{1-\rho}}{1-\rho} = w^*(\theta_1, z_1).$$

Considering multipliers $\eta(\theta_1, z_1)$ for the promise keeping constraints,¹⁹ we have the Lagrangian

$$\sum_{z^2 \in Z^2} \sum_{\theta_1 \in \Theta} \left[\lambda(z^2) \mu_\Theta^1(\theta_1) c_2(\theta_1, z_1, z_2) - \eta(\theta_1, z_1) \mathbb{E}(\theta) \mu_Z^1(z_2) \frac{c_2(\theta_1, z_1, z_2)^{1-\rho}}{1-\rho} \right].$$

Solving the Lagrangian and fixing shadow prices such that the solution respects utility promises and resource constraints generates the result. ■

¹⁹Notice that there are no incentive compatibility constraints in the last period.

Proof. [of Proposition 1] **Part (i):** First, note that total endowment will always be exhausted in period 1, otherwise, utility could be increased by the same amount to every agent without changing incentive constraints. Then, considering $c_1^*(\theta_1, z_1) = \tilde{c}(\theta_1)\eta(z_1)$,

$$\sum_{\theta \in \Theta} \mu(\theta) c_1^*(\theta, z) = z, \forall z \in Z,$$

i.e.,

$$\eta(z_1) = \frac{z}{\sum_{\theta \in \Theta} \mu(\theta) \tilde{c}(\theta_1)}, \forall z_1 \in Z.$$

Then we can redefine \tilde{c} so that $c^*(\theta, z) = z\tilde{c}(\theta)$. Thus, we conclude that separability, in this model, is intrinsically linked to linear functions of endowments.

Consider, then, (c^*, w^*) the candidate optimal consumption plan and promises. Fix a given aggregate endowment $z_1 \in Z$ and only two types $\theta \in (\theta(H), \theta(L))$, define γ and τ such that

$$\mu_{\Theta}^1(L) [w^*(z_1, L) - x]^{\frac{1}{1-\rho}} + \mu_{\Theta}^1(H) [\gamma(x) + w^*(z_1, H)]^{\frac{1}{1-\rho}} = \kappa, \quad (\text{A5})$$

and

$$\theta(L) \frac{[c_1^*(H, z_1) - \varepsilon]^{1-\rho}}{1-\rho} + \quad (\text{A6})$$

$$\beta [w^*(z_1, H) + \gamma(\tau(\varepsilon))] \theta(L) \frac{[c_1^*(L, z_1) + \frac{\mu_{\Theta}^1(H)}{\mu_{\Theta}^1(L)} \varepsilon]^{1-\rho}}{1-\rho} + \quad (\text{A7})$$

$$\beta [w^*(z_1, L) - \tau(\varepsilon)]. \quad (\text{A8})$$

where κ is defined as in the text.

Then, it can be easily checked that

$$\gamma'(x) = \frac{\mu_{\Theta}^1(L)}{\mu_{\Theta}^1(H)} \left[\frac{w^*(z_1, L) - x}{\gamma(x) + w^*(z_1, H)} \right]^{\frac{\rho}{1-\rho}};$$

$$\gamma'(0) = \frac{\mu_{\Theta}^1(L)}{\mu_{\Theta}^1(H)} \left[\frac{w^*(z_1, L)}{w^*(z_1, H)} \right]^{\frac{\rho}{1-\rho}};$$

and that

$$\tau'(\varepsilon) = \frac{1}{\beta [1 + \gamma'(\tau(\varepsilon))]} \left\{ \theta(L) \left[c_1^*(L, z_1) + \frac{\mu(H)}{\mu(L)} \varepsilon \right]^{-\rho} \frac{\mu(H)}{\mu(L)} + \theta(L) [c_1^*(H, z_1) - \varepsilon]^{-\rho} \right\}.$$

Hence,

$$\tau'(0) = \frac{\theta(L)}{\beta [\gamma'(0) + 1]} \left\{ \frac{\mu_{\Theta}^1(H)}{\mu_{\Theta}^1(L)} c_1^*(L, z_1)^{-\rho} + c_1^*(H, z_1)^{-\rho} \right\}.$$

We also know that, from optimality of c^* and proposition 2, that for given z_1 ,

$$0 \in \arg \max_{\varepsilon} \left\{ \mu_{\Theta}^1(L) \left[\frac{\theta(L)}{1-\rho} \left[c_1^*(z_1, \theta(L)) + \frac{\mu_{\Theta}^1(H)}{\mu_{\Theta}^1(L)} \varepsilon \right]^{1-\rho} + \beta [w^*(z_1, L) - \tau(\varepsilon)] \right] + \mu_{\Theta}^1(H) \left[\theta(H) \frac{[c_1^*(z_1, \theta(H)) - \varepsilon]^{1-\rho}}{1-\rho} + \beta [w^*(z_1, \theta(H)) + \gamma(\tau(\varepsilon))] \right] \right\}$$

Therefore, the following first order condition holds for all $z_1 \in Z$,

$$\begin{aligned} & \mu_{\Theta}^1(L) \left[\theta(L) [c_1^*(z_1, L)]^{-\rho} \frac{\mu_{\Theta}^1(H)}{\mu_{\Theta}^1(L)} - \beta \tau'(0) \right] + \\ & \mu_{\Theta}^1(H) \left[-\theta(H) [c_1^*(z_1, H)]^{-\rho} + \beta \gamma'(\tau(0)) \tau'(0) \right] = 0, \end{aligned}$$

that can be re-written as

$$\begin{aligned} & \mu_{\Theta}^1(H) [\theta(L) [c_1^*(z_1, L)]^{-\rho} - \theta(H) [c_1^*(z_1, H)]^{-\rho}] + \\ & \frac{\mu_{\Theta}^1(H) \gamma'(0) - \mu_{\Theta}^1(L)}{1 + \gamma'(0)} \theta^L \left\{ [c_1^*(L, z_1)]^{-\rho} \frac{\mu_{\Theta}^1(H)}{\mu_{\Theta}^1(L)} + [c_1^*(H, z_1)]^{-\rho} \right\} = 0 \end{aligned}$$

$$\frac{\mu_{\Theta}^1(H) \gamma'(0) - \mu_{\Theta}^1(L)}{1 + \gamma'(0)} = \frac{\mu_{\Theta}^1(H) [\theta(H) [c_1^*(z_1, H)]^{-\rho} - \theta(L) [c_1^*(z_1, L)]^{-\rho}]}{\theta^L \left\{ [c_1^*(L, z_1)]^{-\rho} \frac{\mu_{\Theta}^1(H)}{\mu_{\Theta}^1(L)} + [c_1^*(H, z_1)]^{-\rho} \right\}} \quad (\text{A9})$$

Now, suppose that $c_1^*(\theta, z) = z\tilde{c}(\theta)$. Then, the equation above implies

$$\frac{\mu_{\Theta}^1(H) \gamma'(0) - \mu_{\Theta}^1(L)}{1 + \gamma'(0)} = \frac{\mu_{\Theta}^1(H) [\theta(H)\tilde{c}(H)^{-\rho} - \theta(L)\tilde{c}(L)^{-\rho}]}{\theta^L \left\{ \tilde{c}(L)^{-\rho} \frac{\mu_{\Theta}^1(H)}{\mu_{\Theta}^1(L)} + \tilde{c}(H)^{-\rho} \right\}}$$

We can see that

$$\frac{\mu_{\Theta}^1(H) \gamma'(0) - \mu_{\Theta}^1(L)}{1 + \gamma'(0)}$$

should be independent of z . As a consequence,

$$\frac{w^*(z_1, L)}{w^*(z_1, H)}$$

is independent of z , meaning that $w^*(z, \theta) = \xi(\theta) \psi(z)$, but, since

$$\sum_{\theta \in \Theta} \mu_{\Theta}^1(\theta) w^*(z, \theta)^{\frac{1}{1-\rho}} = \kappa, \text{ for all } z \in Z.$$

w^* should be independent of z . Downward incentive compatibility constraints would be slack for every aggregate shock higher than $\min\{z \in Z\}$: a contradiction.

Part (ii) Consider the reallocation presented in the proof of (i). The first order equation is

$$\frac{\mu_{\Theta}^1(H) \gamma'(0) - \mu_{\Theta}^1(L)}{1 + \gamma'(0)} = \frac{\mu_{\Theta}^1(H) [\theta(H) [c_1^*(z_1, H)]^{-\rho} - \theta(L) [c_1^*(z_1, L)]^{-\rho}]}{\theta^L \left\{ [c_1^*(L, z_1)]^{-\rho} \frac{\mu_{\Theta}^1(H)}{\mu_{\Theta}^1(L)} + [c_1^*(H, z_1)]^{-\rho} \right\}}.$$

Consumption in period 2 is totally dependent on period 1 utility promises. Then considering w_1 independent on z_1 , the left hand side of the equation above is independent on z_1 , define ω as this term. This means that

$$[c_1^*(z_1, H)] \theta(H)^{-\frac{1}{\rho}} [\mu(H) - \omega]^{-\frac{1}{\rho}} = [c_1^*(z_1, L)] \left[\theta(L) \mu(H) + \omega \theta(H) \frac{\mu_{\Theta}^1(H)}{\mu_{\Theta}^1(L)} \right]^{-\frac{1}{\rho}}.$$

And therefore first period allocations would be proportional to z_1 . This, together with second period independence on z_1 , is impossible since downward incentive compatibility constraints would become slack when increasing first period shock realization. ■

B. Appendix: Mirrlees' period T allocation

Proof. [of Proposition 3] Part (i): Toward a contradiction, assume that period 2 allocation does not depend on z_1 . Define as $w_t^{z_t}(\tilde{\theta}^t | \theta^t)$ the instantaneous utility at time t , of an agent θ^t that claims to be $\tilde{\theta}^t$ in the aggregate state z^t . From Lemma 2 and the utility functions, it is clear that first period consumption takes the form $c(\theta, z) = \tilde{c}(\theta) \eta(z)$.

Consider the social planner's problem and define $\phi(z)$ as the multiplier of the high-type incentive compatibility in the case of first period shock z . Then, the first order conditions for type θ^H consumption and production are

$$u'(c(H, z)) \left[1 + \frac{\phi(z)}{\mu(z) \mu(H)} \right] = \left[\sum_{\theta} \mu(\theta) u'(c(\theta, z))^{-1} \right]^{-1},$$

and,

$$v' \left(\frac{l_1(H, z)}{z\theta(H)} \right) \frac{1}{z\theta(H)} \left[1 + \frac{\phi(z)}{\mu(z)\mu(H)} \right] = \left[\sum_{\theta} \mu(\theta) u'(c(\theta, z))^{-1} \right]^{-1},$$

respectively.

The first equation, together with 2, implies that $\phi(\cdot)/\mu(\cdot)\mu(H)$, does not depend on z . This means that, considering $v'(y) = y^{\gamma-1}$, $l(\theta, z)$ is of the form $\tilde{l}(\theta)\eta(z)^{\frac{\rho}{\gamma-1}}z^{\frac{\gamma}{\gamma-1}}$. From the binding resource constraint for every aggregate shock, we have that $\eta(z) = z^{\frac{\gamma}{\rho+\gamma-1}}$. The argument above implies that $w_0^z(\theta|\hat{\theta}) = z^{\frac{\gamma(1-\rho)}{\rho+\gamma-1}}w(\theta|\hat{\theta})$, for some function $w(\cdot|\cdot)$. Then, the incentive compatibility constraint becomes

$$z^{\frac{\gamma(1-\rho)}{\rho+\gamma-1}} [w_1(\theta|\theta) - w_1(\theta'|\theta)] \geq \sum_{i=H,L; z_2 \in Z} \mu(i|\theta)\mu(z_2) \left[w_2^{z_2}(\theta', i|\theta, i) - w_2^{z_2}(\theta, i|\theta, i) \right].$$

The high-type incentive compatibility always binds, since, otherwise, it would be welfare increasing to transfer consumption from the high-type to the low-type, who is poorer. Since, by assumption, the right-hand side of the equation above does not depend on z , first period incentives are not being generated through second period allocations, i.e.,

$$\sum_{i=H,L; z_2 \in Z} \mu(i|\theta)\mu(z_2) \left[w_2^{z_2}(\theta', i|\theta, i) - w_2^{z_2}(\theta, i|\theta, i) \right] = 0,$$

for all θ, θ' and z .

Because second period allocations are not contingent on first period reports, the planner's problem can be separated between the two period and the optimal allocation is always the static Mirrlees' problem solution. Define $(c^M(\cdot|z), l^M(\cdot|z))$ as the optimal allocation in the static Mirrlees model with shock z . Lemma 2 implies

$$\frac{c^M(H|z)^{-\rho}}{c^M(L|z)^{-\rho}} = \frac{[\sum_{\theta'} \mu(\theta|H) c^M(\theta|z)^{\rho}]^{-1}}{[\sum_{\theta'} \mu(\theta|L) c^M(\theta|z)^{\rho}]^{-1}},$$

which is true only if $\mu(H|H) = \mu(L|L) = 1$. This is a contradiction.

Part (ii): Here we characterize the last period allocation of a T period version of the model presented in Section 4. Consider the problem being analyzed,

$$\max_{c, y} \sum_{t=1,2} \beta^{t-1} \sum_{\theta^t} \sum_{z^t} \mu_z^1(z^t) \mu_{\Theta}^t(\theta^t) \left[u[c(\theta^t, z^t)] - v \left[\frac{y(\theta^t, z^t)}{\theta_t z_t} \right] \right]$$

subject to

$$\sum_{\theta^t} \mu_{\Theta}^t(\theta^t) [c(\theta^t, z^t) - y(\theta^t, z^t)] \leq 0$$

and

$$\begin{aligned} & \sum_{t=1,2} \beta^{t-1} \sum_{\theta^t} \sum_{z^t} \mu_z^t(z^t) \mu_{\Theta}^t(\theta^t) \left[u [c(\theta^t, z^t)] - v \left[\frac{y(\theta^t, z^t)}{\theta_t z_t} \right] \right] \geq \\ & \sum_{t=1,2} \beta^{t-1} \sum_{\theta^t} \sum_{z^t} \mu_z^t(z^t) \mu_{\Theta}^t(\theta^t) \left[u [c(\sigma(\theta^t, z^t), z^t)] - v \left[\frac{y(\sigma(\theta^t, z^t), z^t)}{\theta_t z_t} \right] \right] \end{aligned}$$

Suppose that the variable of choice is \tilde{c} and \tilde{y} , but that the actual consumption and labor implemented in period T are given by $c_T(\cdot) = \tilde{c}_T(\cdot) z_T^{\frac{\gamma}{\gamma+\rho-1}}$ and $y_T(\cdot) = \tilde{y}_T(\cdot) z_T^{\frac{\gamma}{\gamma+\rho-1}}$, since \tilde{c}_T and \tilde{y}_T also depend on z_T potentially, this change of variable is without loss of generality. Assume that $c_t = \tilde{c}_t$ and $y_t = \tilde{y}_t$ for $t < T$. Given that we are in a finite period model, the incentive compatibility constraint can be written as a restriction that agents do not lie at period t given any history and that they do not plan to lie from t onwards. Then the last period incentive compatibility becomes

$$\begin{aligned} & z_T^{\frac{(1-\rho)\gamma}{\gamma+\rho-1}} \left\{ u [\tilde{c}(\theta^{T-1}, \theta, z^t)] - v \left[\frac{\tilde{y}(\theta^{T-1}, \theta, z^T) z_T^{\frac{\gamma}{\gamma+\rho-1}}}{\theta_T} \right] \right\} \geq \\ & z_T^{\frac{(1-\rho)\gamma}{\gamma+\rho-1}} \left\{ u [\tilde{c}(\theta^{T-1}, \theta', z^t)] - v \left[\frac{\tilde{y}(\theta^{T-1}, \theta', z^T)}{\theta_T} \right] \right\}. \end{aligned}$$

And the resource constraints turns into

$$z_T^{\frac{\gamma}{\gamma+\rho-1}} \sum_{\theta^T} \mu_{\Theta}^T(\theta^T) [\tilde{c}_T(\theta^T, z^T) - \tilde{y}_T(\theta^T, z^T)] \leq 0.$$

Then the constraints on \tilde{c} and \tilde{y} do not depend explicitly on the aggregate shock in the last period. From this and the concavity of the objective function, we get that \tilde{c} and \tilde{y} do not depend on z_T . ■

REFERENCES

- Albanesi, Stefania and Christopher Sleet**, “Dynamic Optimal Taxation with Private Information,” *Review of Economic Studies*, 2006, 73 (1), 1–30.
- Ales, Lawrence and Pricila Maciero**, “Accounting for private information,” Working papers, University of Minnesota 2008.
- Atkeson, Andrew and Robert E. Lucas**, “On Efficient Distribution with Private Information,” *Review of Economic Studies*, 1992, 59, 427–453.

- Chari, Varadarajan V., Patrick J. Kehoe, and Ellen R McGrattan**, “Business Cycle Accounting,” *Econometrica*, May 2007, 75 (3), 781–836.
- , ———, and ———, “New Keynesian Models: Not Yet Useful for Policy Analysis,” *American Economic Journal: Macroeconomics*, 2009, 1 (1), 242–266.
- Golosov, Mikhail, Aleh Tsyvinski, and Iván Werning**, “New Dynamic Public Finance: A User’s Guide,” *NBER Macroeconomics Annual 2006*, 2006.
- Green, Edward**, “Contractual Arrangements for Intertemporal Trade,” in Edward Prescott and Neil Wallace, eds., *Lending and the Smoothing of Uninsurable Income*, Minneapolis: University of Minnesota Press, 1987.
- Hansen, Lars P. and Kenneth Singleton**, “Generalized instrumental variables estimation of nonlinear expectations models,” *Econometrica*, 1982, 50 (5), 1269–1286.
- Holmström, Bengt**, “Moral Hazard in Teams,” *Bell Journal of Economics*, 1982, 10 (0), 74–91.
- Kocherlakota, Narayana**, “Zero Expected Wealth Taxes: A Mirrlees Approach to Dynamic Optimal Taxation,” *Econometrica*, 2005, 73, 1587–1621.
- and **Luigi Pistaferri**, “Household Heterogeneity and Real Exchange Rates,” *Economic Journal*, 2007, 117, C1–C25.
- and ———, “Inequality and Real Exchange Rates,” *Journal of The European Economic Association*, 2008, 6.
- and ———, “Asset pricing implications of Pareto optimality with private information,” *Journal of Political Economy*, 2009, 0, forthcoming.
- Levin, Jonathan**, “Multilateral Contracting and the Employment Relationship,” *Quarterly Journal of Economics*, 2002, 117 (3), 1075–1103.
- Mehra, Rajnish and Edward C. Prescott**, “The equity risk premium: A puzzle,” *Journal of Monetary Economics*, 1985, 15 (2), 145–61.
- Mirrlees, James A.**, “An Exploration in the Theory of Optimal Income Taxation,” *Review of Economic Studies*, 1971, 38, 175–208.
- Phelan, Christopher**, “Incentives and Aggregate Shocks,” *The Review of Economic Studies*, 1994, 61, 681–700.

- Phelan, Christopher**, “On the Long-Run Implications of Repeated Moral Hazard,” *Journal of Economic Theory*, 1998, *79*, 174–191.
- Rogerson, William P.**, “Repeated Moral Hazard,” *Econometrica*, 1985, *53* (1), 69–76.
- Shimer, Robert**, “Convergence in Macroeconomics,” *American Economic Journal: Macroeconomics*, 2009, *1* (1), 280–297.
- Sleet, Christopher**, “Endogenously Incomplete Markets: Macroeconomic Implications,” 2006. Mimeo. Tepper Business School.
- Spear, Stephen E. and Sanjay Srivastava**, “On Repeated Moral Hazard with Discounting,” *Review of Economic Studies*, 1987, *54*, 599–617.
- Thomas, Jonathan and Tim Worrall**, “Income Fluctuation and Asymmetric Information: An Example of a Repeated Principal-Agent Problem.,” *Journal of Economic Theory*, 1990, *51*, 367–390.
- Townsend, Robert**, “Risk and Insurance in Village India,” *Econometrica*, 1994, *62* (3), 539–591.
- Werning, Iván**, “Optimal Fiscal Policy with Redistribution,” *Quarterly Journal of Economics*, 2007, *122*, 925–967.
- Wilson, Robert**, “The Theory of Syndicates,” *Econometrica*, 1968, *36*, 119–132.