Liquidity, Collateral Constraint and Financial Intermediation

Dennis W. Jansen †  Yulei Peng‡
Texas A&M University  University of Arkansas

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Abstract

This paper constructs a general theoretical model to consider two kinds of financial frictions in the economy with financial intermediaries. One is the classical principal-agent problem between financial intermediaries and households, which limits the bankers’ ability to obtain assets indefinitely by borrowing additional funds from households. The other financial friction is a collateral constraint when firms borrow external funds from banks. By formulating households, bankers and entrepreneurs’ decisions in a monetary DSGE framework with nominal rigidities and determining the competitive equilibrium, this model supplies the environment in which we can simulate the production shocks and liquidity shocks to analyze the effects of monetary policies. In a quantitative analysis with three separate shocks, a negative collateral shock, a negative productivity shock and a positive shock to bankers’ diverting rate, this paper finds that a negative collateral shock which tightens firms’ financing constraints on investment can generate an equity price boom which is different from that observed in recessions. This suggests that a collateral shock is not the main driving force of the business cycle, while a negative productivity shock and bankers’ moral hazard problem are important aspects for modeling the economy.

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†Department of Economics, Texas A&M University, College Station, TX 77843. Email: dennis-jansen@tamu.edu. Tel:(979) 845-7358

‡Department of Economics, Sam M. Walton College of Business, Fayetteville, AR 72701. Email:pengyulei@gmail.com. Tel:(479)575-6231
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1 Introduction

In many standard macroeconomic models, identical households can invest in non-financial firms directly, without using a financial sector. As stated in Brunnermeier and Sannikov (2012), this approach can only yield realistic macroeconomic predictions if, in reality, there are no financial frictions. Following the Great Depression, economists such as Fisher (1933) and Keynes (1936) pointed out that failure of financial markets would result in an economic downturn. Financial crises happen every now and then, spilling over into the real economy, and Kindleberger (1993) documents that financial crises have occurred at roughly ten-year intervals in Western Europe over the past four centuries. This means they are relatively common in history. The current financial crisis starting in August 2007 has underscored and reminded us once again of the importance of financial intermediaries for the business cycle.

During the last decade, asset prices have been cyclical and have led the business cycle. Figures 1 and 2 in the Appendix describe the time series of two broad stock price indexes and select macro variables for the US from 1997Q1 to 2012Q2. All series are percentage deviations of quarterly data from trend, and the percentage deviations of investment, GDP and bond prices are rescaled as indicated in the figures.\(^1\) Two phenomena are apparent in these figures. First, investment and real GDP move closely with stock prices. Second, stock prices lead the business cycle. For example, in the 2001-2002 recession, stock prices reached the peak in the first quarter of 2000 and dropped to the trough in the third quarter of 2002, while the investment peaked in the fourth quarter of 2000 and reached the trough in first quarter of 2003. Similarly, in the 2008-2009 recession, stock prices peaked in the third quarter of 2007 before falling to the trough in the first quarter of 2009. Investment and output did not reach the peak until the second quarter of 2008 and did not reach the trough until

\(^1\)There are two kinds of stock price indexes. One is the Wilshire 5000 total market full cap index which is an index of the market value of all stocks actively traded in the US weighted by market capitalization. The designation full cap for an index signifies a float adjusted market capitalization available to ordinary investors. The other index is the S&P 500 index which includes 500 leading companies in leading industries of the U.S. economy. The data for investment is real private nonresidential fixed investment. All the data is available at the Federal Reserve Bank of St.Louis.
the second quarter of 2009.

One intuitive explanation for these patterns is that shocks to the asset market are an important cause of the business cycle instead of merely a response to it. A popular hypothesis along this line is as follows. Sudden drops in equity market liquidity leads equity prices to fall and the price of liquid assets, such as bonds, to rise. When non-financial firms face financing constraints on investment, this reduction in equity price reduces these firms’ ability to finance investment, which will lead to loss of liquidity in the equity markets and a decline in investment and output, thus this is a recession. Kiyotaki and Moore (2012) and Shi (2011) have formulated this hypothesis with a model that places two equity-market frictions at the center. One is the difficulty to issue new equity: a firm can issue new equity to finance at most a fraction of intended investment. The other friction is the lack of ability to resell existing equity. However, issuing new equity is just one channel for a limit number of firms to finance their investment, while a large number of firms borrow the funds they need for intended investment from financial intermediaries. Meanwhile, these firms face a borrowing constraints during the process, as non-financial firms use their market value of capital as collateral.

In this paper, households do not acquire capital and do not provide funds directly to non-financial firms. Instead, they supply funds to banks which act as the liquidity provider, and non-financial firms borrow from banks to satisfy their liquidity demand. In addition to introducing financial intermediaries in the economy, this paper introduces two kinds of financial frictions. The first one is the friction between financial intermediaries and households. As Hellmann et al. (2000) state, moral hazard in the banking sector plays a crucial role in a financial crisis, and abolishing formal deposit insurance systems does not itself solve this agency problem. Since bankers use funds obtained from depositors to make the lending decision, bankers have an incentive to select a more risky asset portfolio which earns high profits if the investment succeeds, but depositors suffer from losses if the investment fails (Kane (1989); Cole et al. (1995)). Therefore, households prefer to limit bankers’ ability to obtain funds.
The other friction is between financial intermediaries and non-financial firms. In the beginning of each period, non-financial firms need to pay wages and get investment funds before they receive the revenue of production. In order to pay for these operating expenses, entrepreneurs who own firms can use the internal funds accumulated in previous periods, and external funds borrowed from banks by using the market value of the firms as collateral. Geanakoplos (2010) points out, in time of crisis, collateral rates are far more important than interest rates, and a shock to the collateral rate will indeed influence the real economy. Gilchrist et al. (2009) empirically show that credit market shocks, which result from deterioration in the supply of credit due to weak balance sheets of firms or the disruptions in the health of banks that supply credit, have played an important role for U.S. business cycle fluctuation during the 1990-2008 period and account for more than 30% of the variation in economic activity as measured by industrial production. Mimir (2010) finds that liabilities and equity are procyclical, the leverage ratio is acyclical and the credit spread is countercyclical. Moreover, drops in stock prices will reduce the non-financial firms’ ability to finance even though they are not receiving the external funds via issuing new equity.

The paper is part of a recent and growing literature which attempts to model the financial intermediaries as an active agent in the economy. Most closely related to my work are Curdia and Woodford (2009), Gertler and Karadi (2011), Gertler and Kiyotaki (2010) and Gertler et al. (2011). Curdia and Woodford (2009) extend the basic New Keynesian Model of the monetary transmission mechanism to allow for a spread between the interest rate available to savers and borrowers, and find that the mere existence of a positive average spread makes little quantitative difference for the predicted effects of particular policies. However, they do not consider the agency problem which is arose by introducing financial intermediaries in the economy. Gertler and Karadi (2011) develop a monetary dynamic stochastic general equilibrium model with financial intermediaries that face endogenously determined balance sheet constraints. Based on Gertler and Karadi (2011), Gertler and Kiyotaki
(2010) and Gertler et al. (2011) consider the financial friction between the bankers and households similar to mine, and explain why banks adopt such a risky balance sheet in the first place. They assume that non-financial firms can only issue new equity to finance investment and they do not consider the financial friction between non-financial firms and banks. In my paper, I regard this existing financial friction which limits non-financial firms' ability to borrow, just as facing a liquidity constraint.

The rationale for liquidity constraints in my model is related to the one in Kiyotaki and Moore (1997) who have offered a theory for how common shocks to credit-constrained firms are amplified through changes in collateral values and transmitted as fluctuations in output. Kiyotaki and Moore (2012) extend the model of a monetary economy where there are differences in liquidity across assets and investigate how aggregate activity and asset prices fluctuate with shocks to productivity and liquidity. Based on this, Shi (2011) finds that for equity price to fall as it typically does in a recession, a negative liquidity shock must be accompanied or caused by other shocks that relax firms' financing constraint on investment. The main distinguishing feature between these models and mine is the assumption that firms financing constraints are exogenous variables and they did not consider the role of financial intermediaries in the economy.

Gorton and Winton (2003) and Brunnermeier et al. (2012) state that financial intermediaries act as liquidity providers in the economy, building upon the Bernanke et al. (1999). They find that the financial frictions between banks and non-financial firms lead to persistence and when combined with illiquidity to non-linear amplification effects. However, they did not consider the problem that can be brought by banks themselves, which is the classical principal-agency problem between bankers and households. This moral hazard problem will limit financial intermediaries' ability to obtain funds from households, which will have impact on the supply side of the market for loanable funds, thereby affect the investment and output.

The basic logic in this paper is as follow: banks are playing an even more important role in the modern economy, especially as liquidity providers for non-financial
firms. While non-financial firms need to borrow from banks to pay the operating expense such as wages and investments, they can use their firms’ market value as collateral to obtain the external funds which can explain the phenomenon between stock prices and investment level. On the other hand, the existence of banks in the economy will give rise to agency problems. The main contribution of this paper is to construct a theoretical model to consider frictions between financial intermediaries and other individuals in the economy. This model formulates households, bankers and entrepreneurs’ decisions in the monetary dynamic stochastic general equilibrium framework with nominal rigidities, and determines the competitive equilibriums. By calibrating the model with a negative collateral shock, a negative productivity shock and a positive shock to bankers’ divert rate, this paper finds that a negative collateral shock which tightens firms’ financing constraints on investment can generate an equity price boom which is different from what is observed in recessions. Therefore, a collateral shock is not the main driving force of the business cycle, while a negative productivity shock and bankers’ moral hazard problem are important aspects to explain current economy. The paper proceeds as follows. In section 2, I outline the environment and describe the equilibrium. Then, I drive the optimal conditions for households, bankers and entrepreneurs, and get the propositions. In section 3, I calibrate the model with three different shocks. In section 4, I conclude.

2 The Model

I consider an infinite-horizon economy in discrete time. The economy is populated by households and non-financial firms, while there exists funds transfer between them. The basic framework is the monetary dynamic stochastic general equilibrium (DSGE) model with nominal rigidities developed by Christiano et al. (2005), Sveen and Weinke (2005) and Smets and Wouters (2007). Basis on this, I add financial intermediaries in the economy and an agency problem constrains the ability of financial intermediaries to obtain funds from households, which is close to the models in (Gertler and Kiy-
otaki, 2010; Gertler et al., 2011). Households do not have investment opportunities besides saving in financial intermediations, and cannot borrow against their future labor incomes, which means that households face a liquidity constraint each period. There is a continuum of firms with unit mass. Each firm produces goods by using a constant returns to scale Cobb-Douglas production function with capital and labor as inputs. Similar to Bernanke et al. (1999) and Fiore et al. (2011), entrepreneurs, who are risk neutral, need to pay the wage and make investment decision in advance to get the revenue from production. In order to finance the liquidity, the entrepreneurs use the accumulated net worth from previous periods and the market value of firms’ capital levels as collateral to borrow the needed funds from financial intermediaries.

If we do not consider financial intermediaries, and if there were no financial frictions in the economy, the competitive equilibrium would be a solution of the social planner’s problem in the standard DSGE model which chooses the series of \( \{C_t, L_t, I_t\} \) as a function of the aggregate state \( \{K_t, A_t\} \) in order to maximize the expected discounted utility of the representative household subject to the resource constraints. In the paper, we use this frictionless DSGE economy as a benchmark to which we may compare the situations with financial frictions.

To be more specific about the timing of events, we decompose each period into three subperiods. In the first subperiod, aggregate shocks and the idiosyncratic collateral ratio shock revealed. Non-financial firms know whether they can re-optimize their price level or not, hire labor supplied by workers and pay their wage bills and collect the funds for investment. They use internal funds brought in from the previous period, and external funds borrowed from financial intermediaries. In the second subperiod, non-financial firms enter into production and sell the outcome of production and buy the goods for investment. Entrepreneurs make consumption decisions and need to pay a tax for their consumption. Households purchase consumption goods and pay lump sum taxes. In the last subperiod, entrepreneurs pay debts to banks and workers make the deposits.

In the following, I will introduce the households, financial intermediaries and
non-financial firms in details and study the consequences for their activities.

2.1 Households

In the economy with financial frictions, households lend to non-financial firms via financial intermediaries. Based on Gertler and Karadi (2011), in period $t$, a representative household with a continuum of members of unit mass, who lives for infinite time periods, chooses aggregate consumption $C_t$ and labor supply $L_t$ in the competitive market, and make deposits in the financial intermediaries. As in Dixit and Stiglitz (1977), the household seeks to maximize the total expected utility

$$E_t \sum_{j=0}^{\infty} \beta^j U(C_{t+j}, L_{t+j}).$$

(1)

Here, $U_C > 0, U_L < 0, U_{CC}, U_{LL} < 0, U_{CL} = 0$, the discount rate $\beta \in (0, 1)$, and $C_t$ denotes the aggregate consumption level at time $t$, which is

$$C_t = \left( \int_0^1 C_t(i)^{\frac{1}{\varepsilon-1}} di \right)^{\frac{\varepsilon-1}{\varepsilon}},$$

where $\varepsilon \geq 1$ is the elasticity of substitution between different kinds of goods $C_t(i)$.

Households take the aggregate price level $P_t$ and the composite goods prices $P_t(i)$ as given and beyond their control. Dealing with the problem of minimizing the expenditure of purchasing $C_t$ units of the composite goods, the household’s decision problem is to:

$$\min \int_0^1 P_t(i)C_t(i)di$$

subject to

$$\left( \int_0^1 C_t(i)^{\frac{1}{\varepsilon-1}} di \right)^{\frac{\varepsilon-1}{\varepsilon}} \geq C_t,$$

(2)
where $P_t(i)$ is the price of good $i$. The demand for good $i$ can then be written as:

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t,$$

where the aggregate price index $P_t = \left( \int_0^1 P_t(i)^{1-\varepsilon} di \right)^{1/1-\varepsilon}$.  

In particular, there are two types of members within each household, which are workers and bankers. Workers supply labor in the competitive labor market and return the wages they earn to the household, while each banker administers a financial intermediary and transfers any nonnegative earnings back to the household. The household thus effectively owns the intermediaries that its bankers manage. However, the deposits that the household holds are put in financial intermediaries that it does not own. This assumption guarantees the bank can make decision independently and the banker maximizes its own net worth other than their utility since the depositors are not the owners of the bank.

At any moment of time $t$, the fraction $1 - f$ of the household members are workers and the remaining fraction $f$ are bankers. A representative household members can switch randomly between these two occupations. A banker this period remains a banker next period with probability $\theta$ which is independent of the banker’s history. Hence, the average survival time of a banker in any given period is $\frac{1}{1-\theta}$, which is finite but may be quite long. Period $t$ bankers learn about survival and exit at the beginning of next period. Each period $(1 - \theta)f$ bankers exit and become workers while the same amount of workers randomly become bankers so that the relative proportion of workers and bankers are constant. Bankers who exit from the financial sector transfer their earnings to their respective household, while the household provides its new bankers with some start-up funds.

While households supply funds to banks rather than acquire capital nor do they

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2The price elasticity of demand for good $i$ is equal to $\varepsilon$. As $\varepsilon \to \infty$, the individual goods become more substitutes, and as a consequence, each firm has less market power.

3The number of workers in a household is far more than the number of bankers, which implies $f < 1 - f$.

4In order to make sure that the bankers do not reach a point where they can finance all equity investment from their own net worth, the bankers are not living with infinitely periods.
provide capital to non-financial firms directly, they just make the deposits in the banks. Since both intermediary deposits and government debt are one period bonds which are both riskless, in the equilibrium we consider, the deposits and government debt are perfect substitutes. The household chooses consumption, labor supply and deposits \( \{C_t, L_t, D_t\} \) to maximize the total expected discounted utility (1) subject to the following sequence of budget constraints:

\[
P_tC_t + D_t = W_tL_t + R^d_tD_{t-1} + \Pi_t - T_t. \tag{4}
\]

Here \( W_t \) is the wage rate, \( R^d_t \) is the nominal interest rate, \( \Pi_t \) is the net distributions from ownership of banks, and \( T_t \) is the lump sum taxes. In each period, since households can not finance their present consumptions from their future labor incomes, they also face the borrowing constraint in each period, which means that:

\[
D_t \geq 0. \tag{5}
\]

The household’s first order conditions for labor supply, consumption and saving are given by:

\[
\frac{W_t}{P_t}U_{C_t} = -U_{L_t}, \tag{6}
\]

\[
U_{C_t} \geq \beta E_t \frac{R^d_t U_{C,t+1}}{1 + \pi_{t+1}} = \beta E_t (1 + r^d_{t+1}) U_{C,t+1}. \tag{7}
\]

Here \( \pi_{t+1} = \frac{P_{t+1}}{P_t} - 1 \) is the inflation rate in period \( t + 1 \), \( 1 + r^d_{t+1} = \frac{R^d_{t+1}}{1 + \pi_{t+1}} \) is the real interest rate, \( U_{C_t} \) denotes the marginal utility of consumption and \( U_{L_t} \) is the marginal dis-utility of labor. Equation (6) states the intratemporal optimal condition, and equation (7) is the Euler equation which can only be equalized as long as the borrowing constraint is not binding. When the constraint is binding, the marginal value of transferring one unit of consumption from period \( t + 1 \) to period \( t \) is positive but cannot be accomplished.

**Assumption 1.** For any \( t \), \( \beta < \frac{1}{1 + r^d_t} \).
Let \( 1 + r_{0,t}^d = \prod_{i=0}^{t}(1 + r_{i}^d) \), within equation (7), for any \( t \), we know that \( \beta^t(1 + r_{0,t}^d)U_{Ct} \) is a bounded supermartingale so we can use Doob’s convergence theorem. If \( \beta(1 + r_{t}^d) < 1 \), which means that the household is relatively impatient given the interest rate, then \( \beta^t(1 + r_{0,t}^d)U_{Ct} \) will convergence, which means that consumption \( C_t \) and deposits \( D_t \) will not diverge. Actually, \( \frac{1}{1+r_t^d} \) is entrepreneurs’ discount rate, if there is no uncertainty in the economy, in the steady state, households will consume their labor income and do not save at all since they are impatient.

2.2 Financial Intermediaries

In each period, the financial intermediaries obtain funds from households and their own net worth which we refer as inside equity. The bank uses all its available funds to make loans to non-financial firms. The balance sheet identity of a financial intermediary at the beginning of period \( t \) which is in the Table 1 is given by:

\[
d_t^s = D_t + n_t,
\]

where \( d_t^s \) denotes the amount of loanable funds supplied by a typical bank and \( n_t \) is the banker’s net worth in the beginning of period \( t \).

<table>
<thead>
<tr>
<th>Loanable Funds ( d_t^s )</th>
<th>Liability ( D_t )</th>
</tr>
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<tbody>
<tr>
<td>Net Equity ( n_t )</td>
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Table 1: A Bank’s Balance Sheet at time \( t \)

The household saves his deposits into bank at the end period \( t - 1 \) and obtains the nominal interest rate \( R_t^d \) at the end of period \( t \), which means \( D_t \) is the liability of the bank and \( n_t \) is its equity. At the end of period \( t \), the bank receives the nominal return rate \( R_t^l \) from its lending. Since \( R_t^l \geq R_t^d \), the bankers always have the incentive to engage in financial intermediaries. The law of motion for the bank’s bet worth is:
Using the balance sheet of banks given by equation (8), we can re-write equation (9) as follows:

\[ n_{t+1} = (R^d_t - R^d_t) d_t + R^d_t n_t. \]  

(10)

The bank’s net worth relies on the premium \( R^d_t - R^d_t \) that earns on the amounts supplied in the loanable funds market.

The profits of the bank will be affected by the premium given above. The banker will not have any incentive to lend out if the return on these loans is less than the cost of deposits. Therefore, the bank will continue to operate in period \( t + j \) if the following inequality is satisfied:

\[ E_t \beta^j \Lambda_{t,t+j} (R^d_{t+j} - R^d_{t+j}) \geq 0, i \geq 0, \]  

(11)

where \( \Lambda_{t,t+j} = \frac{U_{C,t+j}}{U_{C,t}} \). In a perfect financial market, the relation always holds with equality, which means the adjusted premium is zero. In an imperfect capital market, the premium maybe positive due to limits on the intermediary’s ability to obtain funds. Justiniano et al. (2010) supplies the evidence that this premium is highly countercyclical and was widely increased during the recent recession.

The bank’s objective at the beginning of period \( t \) is to maximize the expected discounted terminal net worth, which is

\[ N_t = E_t \sum_{j=0}^{\infty} (1 - \theta)^j \beta^j \Lambda_{t,t+j} n_{t+j}. \]  

(12)

Since the premium \( \beta^j \Lambda_{t,t+j} (R^d_{t+j} - R^d_{t+j}) \) is positive in any period, the bank will always have the incentive to receive more deposits from households. In order to set

\[ 5^5 \text{As the bank faces a financing constraint, it is in its interest to retain all earning until the time it exits, at which point it pays out its accumulated retained earnings as dividends.} \]
a limit on its ability to do so, the economy involves the agency problem. At the end of the period, after obtaining funds from households, the banker can choose to divert \( \varphi \) fraction of assets to the specific household of which the banker is a member. The cost of the banker is that the depositors can force the bank into bankruptcy and recover the remaining fraction \( 1 - \varphi \) of assets. However, it is impossible for the depositors to recover the fraction \( \varphi \) of funds that the banker has already diverted.

Therefore, if households are willing to supply funds to the banker, the following incentive constraint must be satisfied:

\[
N_t \geq \varphi d_t^s. \tag{13}
\]

The left hand side of equation (13) is the value of operating for the bank, which would lose if the banker diverts a fraction of assets. The right hand side of equation (13) is the gain from doing so.

Accordingly, the bank chooses \( d_t^s \) to maximize the expected discounted terminal net worth (12) subject to the law of motion for net worth (10) and the incentive constraint (13).

**Proposition 1.** The expected discounted terminal net worth can be expressed as the discounted total return to its loan to firms and the expected discounted total return to its existing net worth, which means that

\[
N_t = \nu_t d_t^s + \eta_t n_t, \tag{14}
\]

where \( x_{t,t+j} = \frac{d_{t+j}}{d_t} \) and \( z_{t,t+j} = \frac{n_{t+j}}{n_t} \)

\[
\nu_t = (1 - \theta)(R_t^l - R_t^d) + E_t[\theta \beta A_{t,t+1} x_{t,t+1} v_{t+1}],
\]

\[
\eta_t = (1 - \theta) R_t^d + E_t[\theta \beta A_{t,t+1} z_{t,t+1} \eta_{t+1}]. \tag{15}
\]

The proof of the above proposition is contained in Appendix A. In equation (14), \( \nu_t \) can be interpreted as the expected discounted marginal gain to the bank of sup-
plying one more unit of loan, while $\eta_t$ can be regarded as the expected discounted marginal benefit of having one more unit of net worth. As shown in Appendix A, the incentive constraint (13) is always binding which limits the leverage of the bank, thus:

$$d_t^* = \frac{\eta_t n_t}{\varphi - \nu_t}. \tag{16}$$

Combing with equation (8), the bank’s leverage ratio is as follows:

$$\frac{D_t}{n_t} = \frac{\eta_t}{\varphi - \nu_t} - 1. \tag{17}$$

Based on the above equation, we can find that the leverage ratio is an increasing function of the expected marginal return of receiving one more unit of deposits $\nu_t$ and having one more unit of net worth $\eta_t$. Meanwhile, it is also a decreasing function of diverting fraction rate $\varphi$. Intuitively, if the marginal returns increases, it is less attractive for bankers to divert funds today, which will give households more willings to entrust the bankers, and the decreasing of diverting fraction rate will also have the same effect.

Since the fraction $\theta$ of financial intermediaries at time $t$ will survive at the beginning of the next period, the aggregate intermediary net worth $\bar{n}_t$ at period $t + 1$ is the sum of the net worth of surviving financial intermediaries from the previous period and the net worth of newly entering financial intermediaries. While the households provides start-up funds for their respective financial intermediaries, the start-up funds are assumed to be a transfer equal to a fraction of the net worth of exiting bankers, which is $\frac{\epsilon}{1 - \theta}$. Combing with equation (10), (16) and (17), we have the following law of motion for the aggregate intermediary net worth $\bar{n}_{t+1}$:

$$\bar{n}_{t+1} = \theta[(R_t^l - R_t^d)\frac{\eta_t}{\varphi - \nu_t} + R_t^d] \bar{n}_t + \epsilon \bar{n}_t. \tag{18}$$
2.3 Non-financial Firms

The assumptions on the entrepreneurs are as in Bernanke et al. (1999) and Fiore et al. (2011). There is a continuum of monopolistically competitive firms, indexed on the unit interval and owned by specific entrepreneurs. They are risk neutral and have linear preferences over consumption with time preference $\beta^e = \frac{1}{1+r^d}$. Their total expected utility function is:

$$E_t \sum_{j=0}^{\infty} (\beta^e)^j C^e_{t+j}.$$  \hspace{1cm} (19)

Here $C^e_{t+j}$ is the entrepreneur’s aggregate consumption. In each period $t$, entrepreneurs can choose whether use the accumulated internal funds $M_t$ to buy the consumption goods or not. One more unit of consumption at $t$, means that there will be less $1+\tau$ units of real internal funds to be available for production at next period. These funds earn expected return $\frac{R^d_{t+1}}{(1+\pi_{t+1})(1+\tau)}$ units of consumption at $t+1$. Since $R^d_{t+1} > R^d_{t+1}$ and $\beta^e = \frac{1}{1+r^d}$, we have:

$$(1+\tau)\beta^e E_t \frac{R^d_{t+1}}{(1+\pi_{t+1})(1+\tau)} > 1,$$  \hspace{1cm} (20)

then it is better to postpone consumption as they are more patient compared with households. Moreover, the decision does not depend on the consumption tax rate.\textsuperscript{6}

Therefore, the entrepreneur’s object is to maximize the accumulated internal funds which is the market value of the firm.

Each firm $i$ has access to a Cobb-Douglas technology:

$$Y_t(i) = A_t(i)(K_t(i))^\alpha(L_t(i))^{1-\alpha},$$  \hspace{1cm} (21)

where $\alpha \in (0, 1)$ is the capital share in the production function, $A_t(i)$ follows a

\textsuperscript{6}I consider the limiting case where consumption of entrepreneurs is fully taxed. As the tax rate is made arbitrarily large, the consumption of entrepreneurs approaches zero. Actually, I introduce entrepreneurs in the economy and do not want to think of them as actual agents, but as a way to introduce the financial friction that I am interested in.
stationary stochastic process which is common knowledge to all entrepreneurs and realized in the beginning of each period, $K_t(i)$ and $L_t(i)$ denote, respectively, firm $i$’s capital stock and labor input used in its period $t$ production $Y_t(i)$.

Following Woodford (2003), assume that there are two restrictions on capital adjustment. Firstly, the additional capital resulting from an investment decision becomes productive in the next period other than this period. Secondly, firms face a convex capital adjustment cost. This is summarized in the following equation:

$$I_t(i) = f\left(\frac{K_{t+1}(i)}{K_t(i)}\right)K_t(i), \tag{22}$$

where $I_t(i)$ is the amount of goods purchased by firm $i$ at time $t$, and $K_t(i)$ denotes the firm’s capital stock. The function $f(.)$ is assumed to satisfy the following condition: $f(1) = \delta \in (0, 1)$, $f'(1) = 1$, and $f''(1) = \xi > 0$. Parameter $\delta$ is the depreciation rate, and $\xi$ can be treated as the elasticity of the investment to capital ratio with respect to Tobin’s $q$, evaluated in steady state. Moreover, parameter $\xi$ measures the convex capital adjustment cost in a log-linear approximation to the equilibrium dynamics and it is assumed to be strictly larger than zero.

The entrepreneur $i$ joins into period $t$ with capital level $K_t(i)$ and accumulated internal funds $M_t(i)$. As in Kiyotaki and Moore (1997) and Rampini and Viswanathan (2012), the entrepreneur need to pay the wage and make investment decision in advance to receive the revenue from production, and he or she need to use the market value of capital as collateral to borrow external funds from banks. The firm $i$ have the internal funds $M_t(i)$ and the amount it need to borrow is $d_t(i) = W_tL_t(i) + P_tI_t(i) - M_t(i)$, and the collateral constraint:

$$R_l^t(W_tL_t(i) + P_tI_t(i) - M_t(i)) \leq \psi_t(i)E_t[Q_{t+1}(i)K_{t+1}(i)]. \tag{23}$$

Here, $R_l^t$ is the lending interest rate, $Q_{t+1}(i)$ is the market price of capital in currency units, and $\psi_t(i)$ is a collateral shock that reflects the uncertainty in the tightness of

---

7The investment goods is the same as the aggregate consumption bundles.
the credit market. Following Liu et al. (2010), if the entrepreneur fails to pay the
debt, the bank can seize the accumulated capital. Since it is costly to liquidate the
capital stock, the creditor at most would like to lend a fraction \( \psi_t(i) \) of the total
value of collateral assets. The \( \psi_t(i) \) follows the stochastic process:

\[
\log \psi_t(i) = (1 - \rho) \log \bar{\psi}(i) + \rho \log \psi_{t-1}(i) + \epsilon_{\psi t},
\]

where \( \bar{\psi}(i) \) is the average value of \( \psi_t(i) \), \( \rho \in (0, 1) \) is the persistence parameter, and
\( \epsilon_{\psi t} \) is an i.i.d white noise process with mean zero and variance \( \sigma_{\psi t}^2 \).

Assume the staggered price setting as in Calvo (1983) and Yun (1996): each firm
faces a constant and exogenous probability \( 1 - \omega \) of getting to re-optimize its price
in any given period. This structure implies that firm \( i \)'s time \( t \) nominal price \( P_t(i) \)
is either the one that was posted before \( P_{t-1}^*(i) \) with probability \( \omega \) or the optimally
chose price \( P_t^*(i) \) with probability \( 1 - \omega \).

Cost minimization by firms and households implies that the demand for each indi-

gual goods \( i \) in period \( t \) can be written as follows:

\[
Y_t^d(i) = \left( \frac{P_t(i)}{P_t^*} \right)^{-\tau} Y_t^d,
\]

where \( Y_t^d \) is the aggregate demand at time \( t \), which is given by:

\[
Y_t^d = C_t + I_t + G_t,
\]

where \( G_t \) is the government expenditure and \( I_t \equiv \int_0^1 I_t(i) di \) denotes the aggregate
investment demand.

Define \( Q_t K_t(i) = V(K_t(i), M_t(i), P_{t-1}(i)) \), which is the nominal value function
of firm \( i \) joining into period \( t \). Since in the beginning of period \( t \), the entrepreneur
\( i \) has already known whether he can re-optimize or not before borrow the external
funds for banks, he will make different choice when he can control the price level.
Let \( L_t^*(i) \) and \( I_t^*(i) \) respectively be the labor demand and investment level when the
price level is \( P_t^*(i) \). The entrepreneur chooses \( \{L_t(i), K_{t+1}(i), L_t^*(i), K_{t+1}^*(i), P_t^*(i)\} \)
to maximize:

\[
\frac{V(K_t(i), M_t(i), P_{t-1}(i))}{P_t} = \omega \left[ \frac{P_{t-1}(i)Y_t(i)}{P_t} - \frac{R_t^d(i)}{P_t} + \beta^e E_t \frac{V(K_{t+1}(i), M_{t+1}(i), P_{t-1}(i))}{P_{t+1}} \right] \\
+(1 - \omega) \left[ \frac{P_t^*(i)Y_t^*(i)}{P_t} - \frac{R_t^d(i)}{P_t} + \beta^e E_t \frac{V(K_{t+1}(i), M_{t+1}(i), P_{t}^*)}{P_{t+1}} \right],
\]

subject to production function (21), capital motion equation (22), collateral constraint (23) and demand function (25). In equation (26), \(V(K_t(i), M_t(i), P_{t-1}(i))\) denotes the value function in next period if the firm can not re-optimize its price level, while \(V(K_{t+1}(i), M_{t+1}(i), P_{t}^*)\) is the value function if the firm can optimally choose price level. The \(M_{t+1}(i)\) and \(M_{t+1}^*(i)\) are expressed as follows:

\[
M_{t+1}(i) = P_{t-1}(i)Y_t(i) - R_t^d(i), \\
M_{t+1}^*(i) = P_t^*(i)Y_t^*(i) - R_t^d(i).
\]

The detailed steps in solving the firm’s problem are in Appendix B.

**Lemma 1.** When the collateral constraint is binding, the labor demand level is lower than in the frictionless economy.

The existence of collateral constraint will lead the entrepreneur to face more constraints when he makes the decision, and we have:

\[
\frac{P_{t-1}(i)Y_{Lt}(i)}{P_t} \geq \frac{R_t^dW_t}{P_t}, \\
(1 - \frac{1}{\zeta})\frac{P_t^*(i)Y_{Lt}^*(i)}{P_t} \geq \frac{R_t^dW_t}{P_t}.
\]

From the above two inequalities, even though the marginal benefit of adding one unit of labor is no less than the marginal cost, which means that the firm can gain extra profit from increasing the labor input, the firm can not do so because of facing the collateral constraint.

If the collateral constraint is not binding, the first-order condition for price setting
is given by:
\[ \sum_{j=0}^{\infty} (\omega \beta^j) E_t \left\{ \frac{Y_{t+j}(i)}{P_{t+j}} \left[ P_t^*(i) - \frac{\varepsilon}{\varepsilon - 1} MC_{t+j}(i) \right] \right\} = 0, \]  
(28)
where \( MC_t(i) \) denotes the nominal marginal cost of firm \( i \) at period \( t \). By solving firm \( i \)'s cost minimization problem, the expression of \( MC_t(i) \) is:
\[ MC_t(i) = \frac{W_t \ast R_t^d}{Y_{Lt}(i)}, \]
(29)
where \( Y_{Lt}(i) \) is the marginal production of labor. Since the chosen price can be still used as the firms’ future price level, they take into account not only current but also future expected marginal costs. The equation (28) reflects the forward-looking nature of price setting.

### 2.4 Equilibrium

In the economy, the government’s budget constraint is
\[ P_t G_t = M_{t+1} - M_t + T_t, \]
where \( M_t \) denotes the money supply at time \( t \).

For all \( i \in (0, 1) \), given the exogenous process \( \{A_t, \psi_t(i)\}_{t=0}^{\infty} \), the policy processes \( \{T_t\}_{t=0}^{\infty} \), the government spending \( \{G_t\}_{t=0}^{\infty} \), and initial conditions \( \{M_0(i), K_0(i)\} \), a competitive equilibrium of this economy is a collection of \( \{C_t(i), L_t, L_t^*(i), K_t(i), R_t^d, R_t^l, D_t, d_t^s, d_t, I_t(i), M_t, P_t, P_t(i), W_t\}_{t=0}^{\infty} \) such that:

(i) households choose \( \{C_t, L_t, D_t\} \) to maximize the total expected discounted utility subject to the budget constraint and borrowing constraint, taking the prices \( \{P_t(i), P_t, R_t^d, W_t\} \) and the policy processes as given; bankers choose \( \{d_t^s\} \) to maximize the expected discounted terminal net worth subject to the incentive constraint, and entrepreneurs optimize \( \{L_t(i), I_t(i), L_t^*(i), I_t^*(i), P_t^*(i)\} \) to maximize total discount value subject to the collateral constraints by taking \( \{R_t^d, R_t^l\} \) as given.

(ii) All the markets are clear: the labor market \( \int_0^1 L_t(i) di = L_t \); the goods market;
the money $\int_0^1 M_t(i)di = M_t$; the loanable funds $d_t = \int_0^1 d_t(i)di$.

(iii) Government budget constraints hold every period.

(iv) Resource constraint: $Y_t = C_t + I_t + G_t$.

The aggregate price level is an average of the price charged by the fraction $1 - \omega$ of firms setting their price in period $t$ and the average of the remaining fraction $\omega$ of all firms setting their price in earlier periods. However, since the adjusting firms were selected randomly from among all firms, the average price of the non-adjusters is just the average price of all firms that prevailed in previous period. Therefore, as the aggregate price index $P_t = (\int_0^1 P_t(i)^{1-\varepsilon}di)^{\frac{1}{1-\varepsilon}}$, the average price in period $t$ satisfies:

$$P_t^{1-\varepsilon} = (1 - \omega)(P_t^*(i))^{1-\varepsilon} + \omega P_{t-1}^{1-\varepsilon}. \tag{30}$$

Equation (28) and (30) can be approximated around a zero average inflation steady-state equilibrium to obtain an expression for aggregate inflation of the form:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{mc}_t, \tag{31}$$

where $\pi_t = \log(P_t) - \log(P_{t-1})$, $\kappa$ is a parameter which is computed numerically and is an increasing function of the fraction of firms able to adjust each period, nominal marginal cost $mct \equiv \int_0^1 \frac{MC_t(i)}{P_t} di$, and $\hat{mc}_t$ is real marginal cost, expressed as a percentage deviation around its steady-state value. Equation (31) is often referred to as the New Keynesian Phillips Curve, which implies that real marginal cost is the correct driving variable for the inflation process. It also implies that the inflation process is forward-looking as a firm much be concerned with inflation in the future when it sets its price level.

\footnote{See Sveen and Weinke (2005) and Sveen and Weinke (2007) for the detailed calculation process.}
3 Model Analysis

In order to examine how the economy responds collateral shocks and technology shocks, I calibrate the model based on Gertler and Karadi (2011), Kiyotaki and Moore (2012) and Sveen and Weinke (2007).

3.1 Calibration

The quantitative analysis uses the following functional forms for utility and capital adjustment costs:

\[ U(C, L) = \log(C) + \Psi \log(1 - L), \]

\[ f\left(\frac{K_{t+1}(i)}{K_t(i)}\right) = \delta + \frac{K_{t+1}(i)}{K_t(i)} - 1 + \frac{\xi}{2} \left(\frac{K_{t+1}(i)}{K_t(i)} - 1\right)^2. \]

Table 2 lists the choice of parameter values for the baseline model. The quarterly discount factor \( \beta \) is 0.99 which matches the 4% U.S. average annual real interest rate. The relative utility weight of labor is determined so that the steady-state value of \( L \) is 0.33. As estimated in Gali et al. (2001), \( \varepsilon = 11 \) implies a frictionless makeup of 10%. The three financial sector parameters, which are the fraction of capital that can be diverted \( \varphi \), the proportional transfer to new entering bankers \( \epsilon \) and the survival probability \( \theta \), are chosen to match the specific targets, just as stated in Gertler and Karadi (2011).\(^9\) Turning to non-financial firms, the price stickiness parameter \( \omega = 0.779 \) is often considered to be empirically plausible.\(^10\) The parameter pre-multiplying the marginal cost in the inflation equation (31) takes the form \(((1 - \beta^e \omega)(1 - \omega)/\omega)(1 - \alpha)(1 - \alpha + \alpha \varepsilon)\).

In the model, there exists productivity shocks, collateral shocks and bankers’ diverting shocks which I consider separately. The technology \( A_t \) follows the stochastic process:

\[ \log A_t = \rho_a \log(A_{t-1}) + \epsilon_t^a, \]

\(^9\)In the steady-state, the interest rate spread is 100 basis points, and the average survival time of banks is eight years.

\(^10\)Sveen and Weinke (2007) points out that the micro evidence on price adjustment is mixed.
where $\epsilon_t^a \sim N(0, \sigma_a)$. I set the quarterly autoregressive factor $\rho_a$ to 0.96 which is commonly used in the literature, and the standard deviation of shocks is 0.0038. The stochastic process of collateral shock $\psi_t(i)$ is treated as equation (24). Following Liu et al. (2010), the average value of collateral ratio is 0.75, the persistence of collateral ratio shocks is 0.979 and the standard deviation is 0.0126 which are estimated by using the Bayesian method to fit their model to quarterly U.S time series data. Moreover, the bankers’ diverting rate $\varphi$ shock is an $i.i.d$ shock:

$$\log(\varphi_t) = \epsilon_t^\varphi,$$

where $\epsilon_t^\varphi \sim N(0, \sigma_\varphi)$. I choose the standard deviation of the shock $\sigma_\varphi = 0.001713$ so that the value of diverting rate will increase 10% when one unit standard deviation shock happens.

Furthermore, all these shocks are uncorrelated in the model such that we could find out their specific effects.

### 3.2 Experiments

I begin with several experiments designed to illustrate how the model behaves. These figures show the deviations of the variables from their steady state values.

Figure 3 shows the impulse responses to a one-time, one-standard deviation negative shock to collateral ratio $\varphi$. The negative collateral shock increases the investment cost and the interest rate spread, which lead the stock price to rise. On one hand, compared with the steady state, because of the negative collateral shock, the external funds that firms can borrow from banks is lower. On the other hand, as the stock price is higher, it allows the firms’ capital to be more valuable which mitigates the negative effect and the collateral constraint becomes less tight. Furthermore, entrepreneurs spend more expenditure in investment which lowers households’ labor income, even though their labor supply is higher.

Figure 4 presents the impulse responses to a negative shock to productivity. Since
the negative technology shock reduces the marginal productivity of capital, the stock price is lower which makes investment to be less profitable for entrepreneurs. As we can observed from Figure 4, during the first 4 years, stock price, investment and capital level’s deviations from steady state are increasing. Meanwhile, as households can not borrow from the banks, the negative productivity shock lowers the marginal production of labor and real wage, which in turn, decreases the consumption significantly and induces households to supply more labor. Moreover, banks have difficulty in obtaining deposits from households since marginal utility of consumption is higher. This leads a fall in their leverage ratio, which means the productivity shocks generates a procyclical leverage ratio.

Figure 5 shows the impulse response to a positive shock to bankers’ divert rate. If the divert fraction $\phi$ increases, bankers can steal more proportion of money which means that there exists more severe moral hazard problem. In order to convince households to save, bankers have to pay higher interest rate for deposits, which dominates the other effects and rises the leverage ratio. However, since bankers divert funds to their specific households which increases their income, with regarding the income effect, households’ consumption is higher and labor supply is lower temporarily, which reduce the amount of loanable funds in the market and also the investment level. As a result, the production is lower. Without double, the situation with higher consumption level and lower labor supply can not last for a long time which we can find out from the figure.

These responses are broadly consistent with historical data depicted in Figure 1 and Figure 2, although some of the responses do not match in the magnitude. The consistency suggests that productivity shocks are critical aspects to explain the cyclical behavior of stock prices with other macro variables. If productivity shocks are the only shocks, then stock prices would fall and banks’ financing ability becomes worse in response to a negative productivity shock. Meanwhile, positive shocks to bankers’ divert rate can also be used to explain the phenomenons. Even though the reducing in stock prices and outputs is along with higher consumptions, it is mainly
result from the bankers’ diverting. By taking into account this effect, bankers’ moral hazard problem is also important in explaining the cyclical behaviors. Since I have not considered the wage rigidity in the model, the absence of it may imply that labor and output do not fall enough as wage is flexible and can be adjusted without any cost, which may be the reason why labor and output level are increasing after the negative collateral shock.

4 Conclusion

In order to analyze the financial crisis of 2007-2010, Kocherlakota (2010) points out DSGE models need to incorporate both stickiness and financial market frictions. In this paper, I introduce financial intermediaries in the DSGE framework with regarding the nominal rigidities. Accordingly, I consider two kinds of financial frictions between financial intermediaries and other individuals. The first one is the moral hazard problem between financial intermediaries and households as bankers are not using their own funds to make the investment. The other one is the collateral constraint when firms need to borrow external funds from banks. By calibrating the model with three separate shocks which are a negative collateral shock, a negative productivity shock and a positive shock to bankers’ divert rate, I find that compared with the negative collateral shock, the negative productivity shock and bankers’ moral hazard problem are more important aspects. Moreover, it provides a note of caution for policymakers: they should find the reasons of the shortfall in liquidity other than simply pumping liquidity into the market. On one hand, if firms’ shortfall in liquidity is not generated by the fundamental events, it is a good policy for government to supply liquidity to them. If firms’ shortfall in liquidity is generated by productivity, it is not reasonable for government to do so. On the other hand, in order to mitigate bankers’ moral hazard problem, government need to strengthen the supervision of the financial intermediaries, especially in the recession because bankers have more incentive to divert assets during this period.
This paper could be extended into two directions. Firstly, we could use the framework of this paper to have the policy analysis. There are frictions in the capital accumulation process, perhaps related to the intermediation ability of the financial sector. In the financial crisis, the government normally provide the liquidity for banks. Within considering the moral hazard problem, this policy seems a bad idea. Secondly, as financial intermediaries act as liquidity providers in the economy, supplying the microfoundations of the agency problem between bankers, households and entrepreneurs, such as that in Diamond and Dybvig (1983) and Farhi and Tirole (2012), represents a promising avenue for future research.
Appendix

Appendix A: Proof of Proposition 1

Replacing the law of motion for net worth (10) into equation (12), the profit maximization problem by a representative bank is given by

\[ N_t = E_t \sum_{j=0}^{\infty} (1 - \theta)\theta^j \beta^j \Lambda_{t,t+j} [(R_{t+j}^l - R_{t+j}^d) d_{t+j}^s + R_{t+j}^d n_{t+j}] \]

s.t. \( N_t \geq \varphi d_t^s, \)

where \( \mu_t \) is the Lagrange multiplier associated with the incentive compatibility constraint. By using the Lagrangian, the first order conditions are given by

\[ (1 - \theta)\beta \Lambda_{t,t+1}(R_{t+j}^l - R_{t+j}^d) - \mu_t \varphi = 0. \]  \hspace{1cm} (36)

\[ \mu_t (N_t - \varphi d_t^s) = 0, \mu_t \geq 0. \]  \hspace{1cm} (37)

Since, by assumption, the premium \( \beta \Lambda_{t,t+j}(R_{t+j}^l - R_{t+j}^d) \) is positive in any period, from equation (36), we know that \( \mu_t > 0 \), thus the incentive compatibility constraint will hold with equality, which means

\[ N_t = \varphi d_t^s. \]  \hspace{1cm} (38)

Now, let’s write the \( N_t \) in a recursive form and define:

\[ \nu_t d_t^s = E_t \sum_{j=0}^{\infty} (1 - \theta)\theta^j \beta^j \Lambda_{t,t+j} [(R_{t+j}^l - R_{t+j}^d) d_{t+j}^s], \]

\[ \eta_t n_t = E_t \sum_{j=0}^{\infty} (1 - \theta)\theta^j \beta^j \Lambda_{t,t+j} [R_{t+j}^d n_{t+j}]. \]  \hspace{1cm} (39)
Then,
\[ \nu_t = (1 - \theta)(R^d_t - R^d_t) + E_t \sum_{j=1}^{\infty} (1 - \theta)\theta^j \beta^j \Lambda_{t,t+j}\left[(R^d_{t+j} - R^d_{t+j}) \frac{d^s_{t+j}}{d^s_t}\right], \]
\[ \eta_t = (1 - \theta) R^d_t + E_t \sum_{j=1}^{\infty} (1 - \theta)\theta^j \beta^j \Lambda_{t,t+j}\left[R^d_{t+j} \frac{n_{t+j}}{n_t}\right]. \]

Update the above equation one period further, we could have:
\[ \nu_t = (1 - \theta)(R^d_t - R^d_t) + E_t[\theta \beta \Lambda_{t,t+1} x_{t,t+1} v_{t+1}], \]
\[ \eta_t = (1 - \theta) R^d_t + E_t[\theta \beta \Lambda_{t,t+1} z_{t,t+1} \eta_{t+1}], \]
where \( x_{t,t+j} = \frac{d^s_{t+j}}{d^s_t} \) and \( z_{t,t+j} = \frac{n_{t+j}}{n_t} \).

### Appendix B: The firm’s optimization problem

Replace the detail expressions of related variables into the entrepreneur \( i \)'s value function (26). Let \( \chi_t(i) \) be the Lagrange multiplier of collateral constraint when the firm cannot re-optimize the price level, and \( \chi^*_t(i) \) is the Lagrange multiplier associated with collateral constraint when the firm can re-optimize the price level. The first-order conditions are

\[ L_t(i) : \left[\frac{P_t-1(i)Y_{Lt}(i)}{P_t} - \frac{R^d_tW_t}{P_t}\right][1 + \beta e E_t \frac{V_M(t+1)}{1 + \pi_{t+1}}] = \chi_t(i) R^d_t W_t, \]
\[ L^*_t(i) : \left[\frac{(1 - \frac{1}{2})P^*_t(i)Y^*_{Lt}(i)}{P_t} - \frac{R^d_t W_t}{P_t}\right][1 + \beta e E_t \frac{V^*_M(t+1)}{1 + \pi_{t+1}}] = \chi^*_t(i) R^d_t W_t, \]
\[ K_{t+1}(i) : -R^d_t \frac{\partial I_t(i)}{\partial K_{t+1}(i)} + \beta e E_t \frac{V_K(t+1)}{P_{t+1}} - \frac{V_M(t+1)R^d_t}{1 + \pi_{t+1}} \frac{\partial I_t(i)}{\partial K_{t+1}(i)} = \chi_t(i) \left[R^d_t \frac{\partial I_t(i)}{\partial K_{t+1}(i)} - \psi_t(i) E_t Q_{t+1}(i)\right], \]

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By using the Envelope Theorem, we have:

\[
K^*_t(i) = -R_t^l \frac{\partial I_t^r(i)}{\partial K^*_t(i)} + \beta^e E_t \left[ \frac{V^*_K(t+1)}{P_{t+1}} - \frac{V^*_M(t+1)R_t^l}{1 + \pi_{t+1}} \frac{\partial I_t^r(i)}{\partial K^*_t(i)} \right] = \chi_t^*(i)[R^l_t \frac{\partial I_t^r(i)}{\partial K^*_t(i)} - \psi_t(i)E_tQ^*_t(i)].
\] (45)

\[
P^*_t(i) = \frac{(1-\epsilon)Y^*_t(i)}{P_t} + \beta^e E_t \left[ \frac{V^*_M(t+1)}{P_{t+1}} \right] = \beta^e E_t \frac{V_P(t+1)}{P_{t+1}} = 0.
\] (46)

By using the Envelope Theorem, we have:

\[
K_t(i) = \frac{V_K(t)}{P_t} = \omega^{\left(\frac{P_{t-1}(i)Y_{Kt}(i)}{P_t}\right)} \left[ (1 - \omega) \chi_t(i) \left( R_t^l \frac{\partial I_t^r(i)}{\partial K_t(i)} \right) + \beta^e E_t \frac{V_M(t+1)}{P_{t+1}} \right] + \beta^e E_t \left[ \frac{V_K(t+1)(1 - \delta)}{P_{t+1}} \right]
\]

\[
M_t(i) = \frac{V_M(t)}{P_t} = \omega^{\left(\frac{P_{t-1}(i)Y_{Kt}(i)}{P_t}\right)} \left[ (1 - \omega) \chi_t(i) \left( R_t^l \frac{\partial I_t^r(i)}{\partial K_t(i)} \right) + \beta^e E_t \frac{V_M(t+1)}{P_{t+1}} \right] + \beta^e E_t \left[ \frac{V_M(t+1)(1 - \delta)}{P_{t+1}} \right]
\]

\[
P_{t-1}(i) = \frac{V_P(t)}{P_t} = \omega^{\left(\frac{P_{t-1}(i)Y_{Kt}(i)}{P_t}\right)} \left[ (1 - \omega) \chi_t(i) \left( R_t^l \frac{\partial I_t^r(i)}{\partial K_t(i)} \right) + \beta^e E_t \frac{V_M(t+1)Y_t(i) + V_P(t+1)}{P_{t+1}} \right].
\] (49)

Here, \( V_x(t) \) denotes the partial deviation of the value function with respect to \( x \) at period \( t \), \( V^*_x(t) \) denotes the partial deviation of the value function when the firm can optimally choose the price level, \( Y_{Lt}(i) \) is the marginal labor production, \( Y_{Kt}(i) \) is the marginal capital production.

Since \( \chi_t(i) \geq 0 \) and \( \chi^*_t(i) \geq 0 \), within equation (42) and (43), we could have

\[
\frac{P_{t-1}(i)Y_{Lt}(i)}{P_t} \geq \frac{R_t^lW_t}{P_t},
\]

\[
\frac{(1-\frac{1}{\epsilon})P_t^*(i)Y^*_t(i)}{P_t} \geq \frac{R_t^lW_t}{P_t}.
\] (50)
Figure 1: Stock Price and Investment’s Deviation from Trend (%)
Figure 2: Stock Price and GDP’s Deviation from Trend (%)
Table 2: Parameters

<table>
<thead>
<tr>
<th>Preferences</th>
</tr>
</thead>
</table>
| $\beta$ | 0.99 | Discount rate  
| $\Psi$  | 1.63 | Relative utility weight of labor  
| $\varepsilon$ | 11 | Elasticity of substitution  

<table>
<thead>
<tr>
<th>Financial Intermediaries</th>
</tr>
</thead>
</table>
| $\varphi$ | 0.381 | Fraction of capital that can be diverted  
| $\theta$  | 0.972 | Survival rate of bankers  
| $\epsilon$ | 0.002 | Proportional transfer to the new entering bankers  

<table>
<thead>
<tr>
<th>Non-financial Firms</th>
</tr>
</thead>
</table>
| $\alpha$ | 0.36 | Capital share in production function  
| $\omega$  | 0.779 | Probability of keeping prices fixed  
| $\delta$ | 0.025 | Depreciation rate  
| $\xi$  | 3 | Elasticity of the investment to capital ratio  

<table>
<thead>
<tr>
<th>Exogenous Process</th>
</tr>
</thead>
</table>
| $\rho$ | 0.979 | Quarterly persistence of log collateral ratio shocks  
| $\bar{\psi}$  | 0.75 | The average value of collateral ratio  
| $\sigma_\psi$ | 0.0126 | Standard deviation of log collateral ratio shocks  
| $\rho_a$ | 0.95 | Quarterly persistence of log TFP process  
| $\sigma_a$ | 0.0038 | Standard deviation of log TFP shock  
| $\sigma_\varphi$ | 0.001713 | Standard deviation of bankers’ diverting ratio shock  

Figure 3: Impulse responses to a negative collateral shock: deviation from steady-state.
Figure 4: Impulse responses to a negative technology shock: deviation from steady-state.
Figure 5: Impulse responses to a positive diverting shock: deviation from steady-state.
References


