DEMAND REDUCTION IN MULTI-UNIT AUCTIONS WITH VARYING NUMBER OF BIDDERS AND UNITS: EVIDENCE FROM HOMEGROWN VALUE AUCTION EXPERIMENTS

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Abstract

We examine the effect of number of bidders and units on demand reduction effect in the uniform-price auction and the Vickrey auction. We found that increasing the number of bidders increases the first and the second-unit bids in a 2-unit setting and the third and the fourth-unit bids in a 4-unit setting. Increasing the number of units increases the first and the second-unit bids when the number of bidders is lower than the number of units but decreases them when there are more bidders than units. Also, we found that increasing the number of bidders provides significantly higher revenues than those obtained when increasing the number of units.

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1. INTRODUCTION

In recent years, the range of products sold at auctions has expanded considerably to include oil leases, treasury bills, paintings, timber from national forests, cattle, used cars, electromagnetic frequency spectrum, foreign exchange and long term credit (FCC) (Krishna, 2002). Although most of the theoretical work examines the sale of a single object, many of the most important auctions such as in treasury bills involve the simultaneous sale of multiple identical objects; the so called multi-unit auction (Swinkels, 2001).

In multi-unit auctions, the most common mechanisms used in the empirical literature are the uniform-price and the Vickrey auctions. The Vickrey auction is a generalization of the single-unit second price auction. In this mechanism, the winner is paid an amount corresponding to the sum of the bids (not his or her own) that are displaced by his or her successful bids. As the clearing price is not based on winner's bid but on the bids of the other participants, bidding truthfully is a dominant strategy in the Vickrey auction (Engelbrecht-Wiggans et al., 2006)[2]. In spite of its demand-revealing property, the Vickrey auction, however, is not popularly used in real auctions due to two potential drawbacks: 1) the complexity of its pricing rule makes it difficult to understand and implement in the real world; and 2) it is also considered as an unfair-outcome auction as "a low-demand bidder pays more than her high-demand rival, despite the fact that her bids were lower than his" (Engelbrecht-Wiggans et al., 2006).

[2] Due to its demand revealing property, we consider bids obtained in the Vickrey auction as an approximation of individual's true valuations and, therefore, we use it as a reference mechanism.

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In contrast, the uniform-price auction mechanism has been used frequently in Treasury bill and FCC auctions due to its straightforward implementation and, therefore, understandable pricing rule. All winners pay the same price which is equal to the highest rejected bid (Krishna, 2002). Ausubel and Cramton (2002) also mentioned other attractive proprieties of the uniform-price auction such as a fair outcome given that all winners pay the same price. It also greatly widens the market as its pricing rule favors small bidders increasing their probability to be winners. Nevertheless, recent theoretical work (Englebrecht-Wiggans and Kahn (1998) and Ausubel and Cramton (2002)) has demonstrated that uniform price auction entails a potential problem commonly called "demand reduction". In fact, because one of an individual's bids can determine the clearing price (i.e., he or she has to pay for infra-marginal units), the bidder has an incentive to bid less than their values for all units except for the first one, which reduces the seller's revenue and induces economic inefficiencies (Kagel and Levin, 2001). Specifically, Englebrecht-Wiggans and Kahn (1998) (see also: Noussair (1995) and Ausubel and Cramton (2002)) proved in a two-bidder and two-unit environment, that it is a weakly dominant strategy to bid truthfully for the first unit and to deviate from the true valuation for the second unit.

The first part of the theory (demand revealing for the first unit) is demonstrated as follows. Consider v_1^i, b_1^i are ith value and the ith bid for the first unit, respectively, and *p* is the price that the winner has to pay for the first unit. Suppose that $b_1^i < v_1^i$, so if $p > v_1^i > b_1^i$ then bidder i does not win any unit and if $v_1^i > b_1^i > p$, then raising his bid to be equal to v_1^i makes no difference. However, if $v_1^i > p \geq b_1^i$ then bidder i does not win any unit but if he or she bids b_1^i equal to v_1^i then he or she wins the first unit at a profitable price. As we can see, it is a weakly dominant strategy to bid truthfully for the first unit (Krishna, 2002).

Now consider two bidders 1 and 2 whose bids for the first unit are equal to their value v_1^1 and v_1^2 that are distributed according the uniform distribution $[0,1]$, respectively, and let

b_2^1, b_2^2 be their bids for the second unit. Suppose, that for simplicity, bidder 1's value for the second unit $v_2^1 = 0$ (and, as a result, $b_2^1 = 0$) and $b_2^2 \leq v_2^2$. Therefore, if $b_2^2 < v_1^1$ then bidder 2 wins one unit and pays b_2^2; and if $b_2^2 > v_1^1$ then he or she wins two units and pays $b_1^1 = v_1^1$. So from bidding b_2^2, the expected profit of bidder 2 is:

$$\pi(b_2^2) = (v_1^2 - b_2^2)(1 - b_2^2) + \int_0^{b_2^2} 2(v_1^2 - v_1^1) dv_1^1 = v_2^2 - (1 - v_2^2) b_2^2$$

As can be observed, if bidder 2 wants to maximize his or her payoff, he or she has to deviate from his or her true valuation and set it at a value equal to zero (Ausubel and Cramton, 2002)[3].

Dan Levin (2005) proposed another Nash equilibrium for the uniform-price auction[4] that consists of an over bidding for the first unit and an extreme demand reduction for the second unit[5]. As explained in Engelbrecht-Wiggans et al. (2005), the intuition behind this is that when "the bid for the first unit is too high, this makes it impossible for a rival to win a second unit at a profitable price. This then gives the rival an incentive to bid zero on the second unit to keep the price down on the first unit she wins".

Some laboratory and field experiments have been carried out to provide more insights on demand reduction in auctions of two units and two bidders. For example, Wolfram (1998) analyzed bids submitted in daily uniform-price electricity auctions in England and Wales by two firms competing against each other and found evidence of strategic bidding behavior analogous to demand reduction. Kagel and Levin (2001) looked for demand

[3] The demand reduction here is considered extreme, for an alternative demonstration see Krishna (2002).

[4] Engelbrecht-Wiggans et al. (2005) showed that this equilibrium is also applicable to the multi-unit Vickrey auction.

[5] Results from our work and other empirical studies such as List and Lucking-Reiley (2000), Porter and Vragov (2003), Engelbrecht-Wiggans et al. (2006) and Engelmann and Grimm (2009) showed that the majority of second unit bids are positive, which contradicts Levin's prediction that the demand reduction is extreme (second unit bid is equal to zero).

reduction in the uniform price and the Vickrey/Ausubel auctions where two unit bidders competed against robot bidders who demanded only one unit and followed the dominant strategy of bidding their value. As predicted by theory, they observed that bidders sharply reduced their second bids in the uniform price auction and behaved quite sincerely in the Vickrey/Ausubel auction. However, they found that the uniform price auction generated more revenues than the Vickrey/Ausubel auction, which contradicted the theory.

During a sports card show in Orlando, List and Lucking-Reiley (2000) conducted a field experiment offering two identical sports cards for sale at a uniform price and Vickrey auctions. As predicted by Englebrecht-Wiggans and Kahn (1998), they observed systematic bid reduction (as much as 73%) by participants on their second units in uniform price auctions compared to the theoretically demand revealing Vickrey auction. However, subjects' bidding behavior for the first unit failed to meet the theoretical predictions as their bids were higher in the uniform price auction than in the Vickrey auction. Also, they pointed out that the demand reduction did not necessarily lead to reduced incomes to the seller as they did not find significant differences between revenues across auction formats. Porter and Vargov (2006) replicated the List and Reiley's experiments in induced value circumstances and used the English auction as an additional auction mechanism. They observed evidence of demand reduction in the uniform-price and English auctions but found significant overbidding in both the Vickrey and the uniform-price auctions. They therefore concluded that researchers should not rely on the Vickrey auction to obtain estimates of subjects' true values.

Real applications of multi-unit auctions in spectrum auctions have also provided strong evidence of demand reduction. For example, in the FCC's Nationwide Narrowband, PageNet reduced its demand for large licenses from 3 to 2 when it felt that continuing bidding on three licenses will drive up the price on all of the large licenses. On the other hand, in the German auction of third-generation (3G) mobile wireless licenses, Deutsche Telekom continued bidding for three licenses during the 167 rounds of auction and

finished buying two license blocks but paid an extra $2 billion (Ausubel and Cramton, 2002).

Taking all of these into account, we can conclude that demand reduction in uniform price auction is a serious concern that may generate strong inefficiencies and revenue losses. Few theoretical and empirical studies, however, have proposed solutions that could reduce demand reduction effects. Theoretically, Swinkels (2001), within the context of an uncertain total supply in both the discriminatory and uniform-price auction, demonstrated that the demand reduction on price reaches zero when the number of bidders and units is infinitely high. Although using an infinite number of units and bidders in a real multi-unit auction is impossible, it is expected that demand reduction decreases when the number of units and bidders increases. Engelbrecht-Wiggans et al. (2006) showed that for a fixed number of units, the incentives for demand reduction in second unit bids weakly decreased when the number of bidders increased but did not reach zero in the asymptotic limit.

To check the reliability of their theoretical finding, Engelbrecht-Wiggans et al. (2006), following List and Lucking-Reiley (2000), carried out a field experiment using Vickrey and uniform-price auctions. They auctioned two identical units of three types of sports cards with three or five bidders. As expected, they found that demand reduction was indeed present in the treatments of three and five bidders. The demand reduction effects were smaller however than those found in previous empirical work with only two bidders (Wolfram (1998), List and Lucking-Reiley (2000) and Kagel and Levin (2001)). In these experiments, however, Engelbrecht-Wiggans et al. (2006) tested the effect of increasing the number of bidders keeping fixed the number of units at two. But in many real-world auctions such as auctions of Treasury debt, electromagnetic frequency spectrum, electric power etc., the number of units is generally greater than two and, in several cases, even more than the number of bidders.

So it remains to be seen if Engelbrecht-Wiggans et al. (2006)'s results can be extended to Vickrey or uniform-price auctions of more than two units. In fact, in a two-unit setting, participants may reduce their second-unit bid as this bid can later determine the

price that the winner will pay. However, when the number of units is more than two (e.g., 4), we may expect higher second-unit bids as the auction price is now determined by a lower-unit bid (e.g., the fourth-unit bid). Also, previous empirical studies were not able to inform us about the direction of the interaction between the number of units and the number of bidders and what strategy is more effective in reducing demand reduction effects and increasing auction revenues: increasing the number of bidders or increasing the number of units or both. We, therefore, think that information on the responsiveness of buyer's bidding behavior to the variation in the number of bidders and number of units will help auctioneers better control the dimensions of auctions that would generate the best efficiency and outcomes.

Based on the motivation discussed above, we designed an experiment to check the effect on demand reduction of varying both the number of bidders (from 2 to10) and the number of units (from 2 to 4)[6]. In this study, we vary both the number of bidders and units so that we can evaluate the effect on the demand reduction of: (1) increasing the number of bidders keeping the number of units fixed; (2) increasing the number of units fixing the number of bidders; (3) increasing the number of both the bidders and units; and (4) increasing the number of bidders and decreasing the number of units. By increasing the number of bidders and, consequently, competitiveness between participants, we expect a decrease of demand reduction effects, at least, in the second, third and fourth bids. Also, bids may go up when the number of units goes up driving up the probability that each bidder will win.

Another objective of our work is to check if previous results considered either as unpredicted by theory or as anomalies (e.g., first bids were higher in the uniform-price auction than in the Vickrey auction (List and Reiley, 2000) or revenues were higher in the uniform-price auction than in the Vickrey /Ausubel auction (Kagel and Levin, 2001)) – can

[6] Varying the number of bidders and units in both multi-unit Vickrey and uniform-price auctions leads to four treatments: (2_2) two persons_ two units; (2_4) two persons_ four units; (10_2) ten persons_ two units and (10_4) ten persons_ four units.

be generalized or can be considered specific to these studies. Finally, as we deal with up to four units, we extend previous empirical work on demand reduction by studying what happens to the third-unit and the fourth-unit bids when the number of bidders is varied.

We organize the remainder of the article in 4 sections. In the next one, we describe our experimental design. In section 3, we analyze the demand reduction and its sensitivity to the variation in the number of bidders and/or the number of units by comparing results from the Vickrey and the uniform-price auctions. Specifically, we analyze the mean of first-unit bids, the mean of second-unit bids, the mean of the third-unit bids, the mean of the fourth-unit bids, the proportion of zero bids (extreme demand reduction), individual bid schedules (difference between the bids of the same subject) and auctions revenues. In section 4, we draw some concluding remarks

2. EXPERIMENTAL DESIGN

We designed an experiment to investigate the effect of varying the number of bidders and units on demand reduction in multi-unit auctions. In June 2008, we conducted an equal number of uniform-price and Vickrey auctions on identical packets (40g) of a product (i.e., organic chips). 160 undergraduate students were recruited and were randomly assigned, equally, to four treatments. In both mechanisms, we carried out ten sessions of two bidders and two units (2_2), ten sessions of two bidders and four units (2_4), two sessions of ten bidders and two units (10_2) and two sessions of ten bidders and four units (10_4). No subject participated in more than one session. as we conducted the experiment through five rounds and needed to determine the winner(s) and the price to be paid after each round, we executed the experiment in a computer lab using the z-tree software (Fischbacher, 2007) that allowed us to collect participants' bids and to compute all needed information in a relatively short time. Table 1 shows the number of auctions ran during the experiment. In total, we ran 48 auctions, consisting of 24 uniform-price auctions and 24 Vickrey auctions.

We conducted the experiment using a four-step procedure. In step 1, participants were invited to a specific computer lab at a specific day and hour. Each person sat in a table separated from the rest to minimize any possible interactions and so that each participant would report his or her bid in an anonymous manner. After taking a seat and given a welcome address, each participant received an envelope which contained 10 Euros as compensation for their participation, his or her identification number (to be held in secret during the process) and a questionnaire designed to collect information on participants' socio-demographic characteristics.

In step 2, once the questionnaire was completed, the actual experiment began. One of the determinants of success of experiments is a good understanding by the participants of the operating procedures used in the auction mechanism. To achieve this goal, we gave each participant a printed material that included an explanation of how auction works and some examples to illustrate the auction[7]. The instructions were identical across all treatments except for auction type and number of bidders and units. After reading and discussing the instructions, participants were given the opportunity to ask questions to dissipate any doubts about the process. Finally, to permit a better understanding of the auction mechanism and a good familiarity with the software, we carried out a training session using an auction of two or four (depending on the treatment) identical packets of the product we used in the auctions (i.e, organic chips), informing participants that no actual economic exchange will take place at the end of the training session.

In step 3 and once the participants became familiar with the procedure, the subjects were requested to come close to a table where a few packets of organic chips are placed along. Each participant was allowed to visually and manually inspect the auctioned product.

[7] The printed material used in our experiment has been prepared based on the instructions' sheet used in Engelbrecht-Wiggans et al. (2006).

Once all participants have finished inspecting the product, the auction began. Each participant had to submit, via the computer, how much he or she was willing-to-pay for each auctioned unit. Once all participants finished to report their bid, the software displayed whether the participant was the winner or not and the price that he/she had to pay for each unit won. The same process was repeated for four additional rounds.

In step 4, the auctioneer closed the auction (after the fifth round) and one binding round was chosen randomly to determine the winner(s). The winner(s) of the binding round was (were) appointed as the winner(s) of the auction. Once the results were announced, the experiment ended by handing the product to the winner(s) who had to pay the corresponding market-clearing price.

3. RESULTS

As mentioned previously, the main objective of our study is to assess the effect of the variation in the number of bidders and units on demand reduction. Specifically, we examined the first, second, third and fourth unit bids, zero bids, bid schedules, and revenues to test the effects on demand reduction. To put our results into perspective, we begin each part by presenting the theoretical prediction based generally on the work of Engelbrecht-Wiggans and Khan (1998) and Engelbrecht-Wiggans et al. (2006). We then report previous empirical findings based on the studies of Kagel and Levin (2000), List and Reily (2000), Engelbrecht-Wiggans et al. (2006) and Porter and Vargov (2006), along with the results of our experiments.

First-unit bid

As mentioned in the introduction, the theory predicts that bidders truthfully report their bids for the first auctioned unit in both the Vickrey and uniform-price auction mechanisms. Therefore, we expected no difference in the first-unit bids between the Vickrey auction and those in the uniform-price auction. Empirically, within the context of two bidders and two

units, List and Reily (2000), unexpectedly found significant and higher first-unit bids in uniform-price than in Vickrey auctions. This violation of the theoretical prediction, however, disappeared as the number of subjects increased (Engelbrecht-Wiggans et al, 2006). In an induced-value context, Porter and Vargov (2006) explained the List and Reily's (2000) unexpected overbidding behavior results and concluded that both Vickrey and uniform-price auctions were not demand revealing.

Our results for the first-unit bids are displayed in the second column in Table 2. Contrary to previous empirical findings, our results showed that the mean of first-unit bids is significantly higher in the Vickrey auctions than in the uniform-price auctions in all treatments except in the 10_2 treatment, sending an initial signal of a possible demand reduction in first-unit bids. The reason for this result is unclear. However, it is possible that the high level of competitiveness in the 10_2 treatment could have caused the significantly higher first-unit bids in both Vickrey and uniform-price auctions, compared to those in the other treatments.

In theory, Englebrecht-Wiggans et al. (2006) predicts that the mean of the first unit bids should be independent of the number of bidders in both the Vickrey and the uniform-price auctions. Results from their experiment showed an insignificant change in participants' first-unit bids when the number of bidders increased from 3 to 5. In our experiment, we checked the sensitivity of first-unit bids to the variation of both participants' number (from 2 to 10) and units' number (from 2 to 4). Data in the third column of Table 3 suggest that in both mechanisms, the mean of first-unit bids increases significantly when the number of bidders is increased from 2 to 10, keeping the number of units constant at 2. However, this increase is less when the number of auctioned units is doubled to 4.

In terms of the effect of number of units (*n*) on participants' first-unit bids, we found that the increase in n (setting the number of bidders at 2) positively affected the mean of the first-unit bids in both mechanisms but this increase is statistically significant only in the uniform-price auction. However, the effect is significantly negative when this increase is implemented

in groups of 10 bidders, moving the bids from 1.31 to 0.99€ (1.26 to 0.85€) in the Vickrey auction (uniform-price auction). Finally, our results (line 5, 6, 11 and 12 of column 3) show that carrying out our auctions with a high number of bidders and a low number of units (10_2) yields significantly higher first-unit bids than doing it within the context of more units than bidders (2_4). Also in both mechanisms, we found that increasing both bidders' and units' number (10_4) generated higher bids for the first-auctioned unit than those obtained in an environment of 2 bidders and 2 units.

In general, participants' bidding behavior in the Vickrey and the uniform-price auctions seems to be controlled by a specific trade off between the number of bidders and the number of units. In a 2 unit (bidder) setting, the rise in the number of bidders (units) increases the mean of first-unit bids. But if the increase in the number of bidders (units) is implemented within the context of a higher number of units (bidders), then the positive effect is inhibited in the former scenario and converted to a negative effect in the second one.

Second-unit bids

Within the context of a 2-bidder, 2-unit setting, theoretical studies by Noussair (1995), Englebrecht-Wiggans and Kahn (1998) and Ausubel and Cramton (2002) predicted lower second unit bids in uniform-price auctions than in Vickrey auctions. Also, Kagel and Levin (2005) and Engelbrecht-Wiggans et al. (2006) demonstrated that the demand reduction effects on mean second-unit bids in the uniform-price auction decreases as the number of bidders increase beyond 2. Engelbrecht-Wiggans et al. (2006)'s empirical findings were consistent with their theory but the difference between treatments was not significant. Our results in columns 5 and 6 of Table 2 generally provide support for the theoretical prediction. In fact, in the four treatments, the mean of second-unit bids is significantly lower in uniform-price auctions than in Vickrey auctions. Nonetheless, increasing the number of bidders and (or) units seems to be ineffective in significantly decreasing the difference between the two mechanisms except in the 10_2 treatment where we

unexpectedly observed an increase in the difference.

The effect of varying the number of bidders and units on second-unit bids in Vickrey and uniform-price auctions seems to be also governed by the same trade off that ruled participants' bidding behavior for the first unit. Taking as a base the 2 units and 2 bidders scenario, columns 5 and 6 of Table 3 show that an increase in the number of bidders (units) from 2 to 10 (2 to 4) led to a significant increase in the average of the second-unit bids in both Vickrey [from 0.60€ to 0.98€ (0.60€ to 0.78€)] and uniform-price [from 0.47€ to 0.68€ (0.47€ to 0.66€)] auctions. However, keeping fixed the number of units (bidders) at 4 (10) and increasing the number of bidders (units) from 2 to 10 (from 2 to 4) did not significantly change (decreased) participants' second-unit bids in both mechanisms (being significant only in the Vickrey auctions). In addition, our results (lines 5,6,11 and 12 of column 4) show that increasing bidders' number generates significantly higher second-unit bids than increasing units' number (0.98€ versus 0.78€) in the Vickrey auctions, but no difference was observed in the uniform-price auctions. We also found that jointly increasing the number of bidders and units yielded significantly higher second-unit bids in both mechanisms than those observed in a 2-bidder and 2-unit setting (0.81€ versus 0.60€ in the Vickrey auction and 0.64€ versus 0.47€ in the uniform-price auction).

Overall, our results suggested three[8] strategies that can be used to diminish demand reduction effects in uniform-price auctions, in relation to results obtained with an equivalent number of bidders and units (2 bidders and 2 units): 1) increasing the number of bidders, 2) increasing the number of units; and 3) increasing both the number of bidders and units. Also, it is important to note that in the Vickrey auctions, our findings diverged from the theory that predicted no change in second-unit bids in response to an increase in the number of bidders, putting in doubt its demand revealing character. In fact, participants in the Vickrey auctions responded positively and significantly to an increase in the number of competitors or/and the

[8] These strategies are relatively equivalent in generating similar second-unit bid values.

number of auctioned units.[9]

Third and fourth-unit bids

Unlike previous empirical work, our experimental design allowed us to check the sensitivity of the third and the fourth-unit bids to a variation in the number of bidders. As expected, the mean of the third-unit bids and the mean of the fourth-units bids in treatments 2_4 and 10_4 were significantly lower in the uniform-price auctions than in the Vickrey auctions (column 4 and 5 in Table 2). We noted also that the difference between bids in the Vickrey auctions and bids in the uniform-price auctions (and therefore the demand reduction) is increasing in the number of units, reaching its maximum in the fourth-unit bids. Unlike the first and the second-unit bids, increasing the number of bidders in a 4-unit auction setting significantly increases the mean of the third-unit bids, in the Vickrey auctions, and the mean of the fourth-unit bids, in both mechanisms (column 7 to 10 in Table 3). It seems that the increase in the number of bidders positively affects bids that determine the price the winner has to pay. In fact in a two-unit setting, the increase in the number of bidders significantly increases the mean of second-unit bids. However, when we auction four units, the increase in the number of bidders does not affect the second-unit bids, but it increases the fourth-unit bids in both mechanisms.

Zero bids

Assessing the proportion of zero bids (full demand reduction) is another way to study demand reduction in the Vickrey and the uniform-price auctions. Engelbrecht-Wiggans and Khan (1998) and Engelbrecht-Wiggans et al. (2006) theoretically predicted a higher proportion of zero bids on second units in the uniform-price auctions than in the Vickrey auctions and a similar proportion of zero bids on first units in both mechanisms.

[9] To check for a possible learning effect through rounds, we carried out the same analysis in each round. We found that participants' bidding behavior is similar in all rounds. In general, there is an increase in bids through rounds, being significant only between round 1 and round 5 and between round 3 and round 5 in the fourth bid.

Empirically, List and Reily (2000) found evidence of more zero bids on the second units in uniform-price auctions in the treatments with more experienced bidders. Engelbrecht-Wiggans et al. (2006) found higher proportion of zero bids on the second units in the uniform-price auction than in the Vickrey auction, but this difference was not statistically significant in all treatments. Our results (Table 4) show that as predicted by the theory, the proportion of zero bids was higher in the uniform-price auction than in the Vickrey auction. Generally, no significant difference was found in both the first-unit bids and the second-unit bids (except in treatments 10_4 (first-unit bids) and 2_2 (second-unit bids)). However, the proportion of zero bids on the third and the fourth units increases significantly when we move from the Vickrey to the uniform-price auctions. The demand reduction tends to be extreme as the number of auctioned units increases. In fact 55% of bids on the fourth unit in the uniform-price auction were equal to zero. Like in previous empirical works, the Levin equilibrium, which predicted zero bids on all units beyond the first one, seems to be not the dominant strategy being followed by the majority of participants in our experiment as the greater part of bids for the second, the third and the fourth units are non-zero.

Regarding the effect of varying the number of bidders and units, Engelbrecht-Wiggans et al. (2006)'s theory predicts that the increase in the number of bidders would not affect the proportion of zero bids in the Vickrey auctions but would decrease it weakly in the uniform-price auctions. In their experiment, the proportion of zero bids decreased in the uniform-price auction in response to an increase in the bidders' number (from 3 to 5) but this decrease was insignificant. Our results (Table 5) show that increasing the number of bidders and /or the number of units significantly affects the proportion of zero bids only on the second-units. The direction of this effect was opposite that predicted by Engelbrecht-Wiggans et al. (2006). In fact, the proportion of zero second-unit bids increased when the number of bidders increased and this proportion decreased when the number of units increased. This bidding behavior can perhaps be explained by the change in the probability of winning. When the number of bidders increases, the probability of

winning the second unit decreases and, as a result, some bidders may focus on winning the first unit and reduce their demand for the second unit to maximize their profit and the opposite scenario occurs when we move up the number of units. Contrary to the theoretical prediction, our results on zero bids confirmed the sensitivity of the Vickrey auction to the variation of the number of bidders and units and, therefore, support the conclusion of Porter and Vargov (2006) who suggested abandoning the idea that the Vickrey auction is a demand revealing mechanism.

Bid schedules

Another test of demand reduction relates to analyzing the slope of the bid schedules submitted by individual bidders. By bid schedules, we mean the difference between individual's first-unit bids and her or his second-unit bids, individual's second-unit bids and his or her third-unit bids; and individual's third-unit bids and his or her fourth-unit bids. Based on the theoretical background and on previous empirical results that showed the higher sensitivity of the uniform-price auctions to demand reduction, it is expected that the difference will be greater in the uniform-price auctions than in the Vickrey auctions. Data in Table 6 show that bidders' bid schedules are, generally, more sloped in the uniform-price auctions, being significant only in the difference between individual's first-unit bids and his or her second-unit bids in treatments 10_2 and 2_2 and between individual's second-unit bids and her or his third-unit bids in treatments 10_4 and 2_4.

In relation to the effect of the number of bidders and units on bid schedules, results in Table 7 show that as the number of bidders increases, the differences between individual's second-unit bids and individual's third-unit bids and between individual's third-unit bids and individual's fourth-unit bids, decrease significantly in both mechanisms. However, the difference between individual's first unit and second unit bids decreases significantly only when the number of units increases, especially in a setting of more bidders than units.

Next, we examine the proportion of flat bid schedules, that is, the proportion of first-unit bids equal to second-unit bids, the proportion of second-unit bids equal to third-unit bids and the proportion of third-unit bids equal to fourth-unit bids. Demand reduction theory predicts that the proportion of flat-bid schedules should be higher in the Vickrey auctions than in the uniform-price auctions (List and Reily, 2000). Results reported in Table 8, generally, confirm this prediction but the difference between the proportions of flat-bid schedules in the two mechanisms is not significant (except for the proportion the third-unit bids equal to fourth-unit bids in the treatment 2_4). As exhibited in Table 9, flat-bid schedules, in general, are affected positively by the increase in the number of bidders but this effect is statistically insignificant (except for the proportion of the third-unit bids equal to fourth-unit bids in the treatment 2_4 in the Vickrey auction). However, as the number of units increases, the proportion of flat-bid schedules increases significantly in both mechanisms (except in the uniform-price auction where the increase is significant only in the case of 2 bidders).

Revenues

In real word auctions, revenue is sometimes used as a selection criterion between auction mechanisms especially for sellers. Theoretically, a definitive revenue ranking of multi-unit auction mechanisms has not yet been established due to, among others, the significant dependence of this ranking criterion on the underlying demand structure (Ausubel and Cramton, 2002). Empirically, Kagel and Levin (2000), unexpectedly, found higher revenues in the uniform-price auctions than in the Vickrey auctions. List and Lucking-Reiley (2000) and Engelbrecht-Wiggans et al. (2006) did not find significant revenue difference between both mechanisms. In our experiment, the Vickrey auctions yielded higher revenues, but this difference was not significant (Table 10).

Although the role of revenue in ranking auctions is still ambiguous, it can be a powerful criterion for sellers to use in choosing between increasing the number of bidders

or the number of units. In fact, as reported in Table 11, the Vickrey or uniform-price auctions with 10 bidders yielded significantly higher revenues (more than threefold) than the auctions with only 2 bidders. However, varying the number of units seems to be ineffective. Therefore, as in single-unit auctions, increasing the competitiveness between bidders (increasing the number of participants) in multi-unit auctions can also provide auctioneers higher revenues. On the other hand, we have seen in part 3.2 that the auctioneer should be indifferent between increasing the number of bidders or the number of units in reducing demand reduction effects in the second unit. However, taking into account the last result, increasing the number of bidders not only reduces demand reduction but also generates more revenues compared to increasing the number of units.

4. CONCLUSION

In the last few decades, multi-unit auctions have been increasingly used in the transaction of several goods (treasury bills, oil leases, spectrum rights etc.). The implementation of multi-unit auction in practice is, however, influenced by the problem of demand reduction which can dramatically affect the allocation of the good and the outcome of the auction. Previous theoretical work (e.g., Swinkels 2001) showed that large multi-unit auctions (infinitely high number of bidders and units) are less sensitive to demand reduction. Empirically, Engelbrecht-Wiggans et al. (2006) tested the effect of number of bidders on demand reduction effects using 3 to 5 bidders. In this study, we analyze the sensitivity of demand reduction to a relatively “large” number of bidders and/or relatively “large” number of units.

Our analysis suggested a number of important points. First, consistent with previous empirical studies, we found that the uniform-price auction is more sensitive to demand reduction than the Vickrey auction. Departing from a setting of equal number of units and bidders (2 bidders and 2 units), our results suggest that an increase in the number of bidders or the number of units decreases demand reduction in uniform-price auction. Therefore, it

seems that running uniform-price auctions with large number of bidders and units can be useful in reducing demand reduction effects. Second, we found that bidding truthfully for the first unit and zero for the rest of the units is more evident in uniform-price auctions, not being the dominant bidding strategy as predicted by Levin (2005). Our result also showed that extreme demand reduction in second-unit bids is sensitive to the probability of winning. In fact, it decreases as the number of units increases and becomes larger when the number of bidders increases. Third, we found that the difference between participant's bids is more evident in uniform-price auctions. Specifically, the difference between participant's first-unit bids and second-unit bids is decreased by an increase in the number of units. However, the difference between participant's second-unit bids and third-unit bids and the difference between participant's third-unit bids and fourth–unit bids are decreased with an increase in the number of bidders. Hence, in general, increasing the number of bidders and/or the number of units decreases demand reduction in the uniform-price auction but does not eliminate it. Finally, we found that increasing the number of bidders generates more revenue than increasing the number of units.

Although our results showed that demand reduction is more evident in uniform price auction, Vickrey auction has proved to be notably sensitive to variation in the number of bidders and/or units, putting in doubt its demand revealing property. However, as we carried a homegrown value experiment, we were not able to investigate for a possible over or underbidding behavior in Vickrey auctions. Hence, future research on the effect of the number of bidders and/or the number of units on demand reduction in an induced value setting is warranted.

TABLE 1- EXPERIMENTAL TREATMENTS

Treatment	Bidders per auction	Units per auction	Vickrey auction sessions	Uniform price auction sessions	Total subjects
2_2	2	2	10	10	40
2_4	2	4	10	10	40
10_2	10	2	2	2	40
10_4	10	4	2	2	40

TABLE 2 - AVERAGE BIDS

TREATMENT	BID1			BID2			BID3			BID4		
	VICKREY	UNIFORM	p-value	VICKREY	UNIFORM	p-value	VICKREY	UNIFORM	p-value	VICKREY	UNIFORM	p-value
10_2	1.31	1.26	0.30	0.98	0.68	**0.00**	-	-	-	-	-	-
10_4	0.99	0.85	**0.00**	0.81	0.64	**0.00**	0.64	0.40	**0.00**	0.54	0.29	**0.00**
2_2	0.91	0.66	**0.00**	0.60	0.47	**0.00**	-	-	-	-	-	-
2_4	0.96	0.85	**0.04**	0.78	0.66	**0.01**	0.54	0.34	**0.00**	0.36	0.18	**0.00**

TABLE 3 - THE EFFECT OF VARYING THE NUMBER OF BIDDERS AND THE NUMBER OF UNITS ON BIDS

AUCTION FORMAT	TREATMENTS	BID1	p-value	BID2	p-value	BID3	p-value	BID4	p-value
VICKREY	2_2 to 10_2	0.91 to 1.33	**0.00**	0.60 to 0.99	**0.00**	-	-	-	-
	2_4 to 10_4	0.96 to 0.99	0.33	0.78 to 0.81	0.22	0.54 to 0.64	**0.01**	0.36 to 0.54	**0.00**
	2_2 to 2_4	0.91 to 0.96	0.80	0.60 to 0.78	**0.00**	-	-	-	-
	10_2 to 10_4	1.33 to 0.99	**0.00**	0.99 to 0.81	**0.02**	-	-	-	-
	2_4 to 10_2	0.96 to 1.33	**0.00**	0.78 to 0.99	**0.00**	-	-	-	-
	2_2 to 10_4	0.91 to 099	0.08	0.60 to 0.81	**0.00**	-	-	-	-
UNIFORM	2_2 to 10_2	0.66 to 1.26	**0.00**	0.47 to 0.68	**0.00**	-	-	-	-
	2_4 to 10_4	0.85 to 0.85	0.50	0.66 to 0.64	0.35	0.34 to 0.40	0.15	0.18 to 0.29	**0.00**
	2_2 to 2_4	0.66 to 0.85	**0.00**	0.47 to 0.66	**0.00**	-	-	-	-
	10_2 to 10_4	1.26 to 0.85	**0.00**	0.68 to 0.64	0.28	-	-	-	-
	2_4 to 10_2	0.85 to 1.26	**0.00**	0.66 to 0.68	0.38	-	-	-	-
	2_2 to 10_4	0.66 to 0.85	**0.00**	0.47 to 0.64	**0.00**	-	-	-	-

TABLE 4 - PROPORTION OF ZERO BIDS (%)

Treatment	Zero first-unit bid		p-value	Zero second-unit bid		p-value	Zero third-unit bid		p-value	Zero fourth-unit bid		p-value
	Vickrey	Uniform		Vickrey	Uniform		Vickrey	Uniform		Vickrey	Uniform	
10_2	0	1	0.32	20	31	0.07	-	-	-	-	-	-
10_4	0	4	**0.04**	5	9	0.27	14	34	**0.00**	22	48	**0.00**
2_2	2	2	1.00	18	7	**0.02**	-	-	-	-	-	-
2_4	0	0	1.00	0	2	0.16	4	31	**0.00**	18	55	**0.00**

TABLE 5 - THE EFFECT OF VARYING THE NUMBER OF BIDDERS AND THE NUMBER OF UNITS ON THE PROPORTION OF ZERO BIDS

Zero Bids		2_2	10_2	p-value	2_4	10_4	p-value	2_2	2_4	p-value	10_2	10_4	p-value
Zero first-unit bid	Vickrey	2	0	0.15	0	0	1.00	2	0	0.15	0	0	1.00
	Uniform	2	1	0.56	0	4	**0.04**	2	0	0.15	1	4	0.17
Zero second-unit bid	Vickrey	18	20	0.71	0	5	**0.02**	18	0	**0.00**	20	5	**0.00**
	Uniform	7	31	**0.00**	2	9	**0.03**	7	2	**0.08**	31	9	**0.00**
Zero third-unit bid	Vickrey	-	-	-	4	14	**0.01**	-	-	-	-	-	-
	Uniform	-	-	-	31	34	0.65	-	-	-	-	-	-
Zero fourth-unit bid	Vickrey	-	-	-	18	22	0.48	-	-	-	-	-	-
	Uniform	-	-	-	55	48	0.32	-	-	-	-	-	-

TABLE 6 - INDIVIDUAL BID SCHEDULES

Treatment	Bid1-Bid2		p-value	Bid2-Bid3		p-value	Bid3-Bid4		p-value
	Vickrey	Uniform		Vickrey	Uniform		Vickrey	Uniform	
10_2	0.33	0.58	**0.00**	-	-	-	-	-	-
10_4	0.18	0.21	0.16	0.17	0.24	**0.04**	0.10	0.11	0.44
2_2	0.30	0.19	**0.00**	-	-	-	-	-	-
2_4	0.19	0.19	0.48	0.24	0.31	**0.03**	0.18	0.12	0.29

TABLE 7 - THE EFFECT OF VARYING THE NUMBER OF BIDDERS AND THE NUMBER OF UNITS ON INDIVIDUAL BID SCHEDULES

Bid schedules		2_2	10_2	p-value	2_4	10_4	p-value	2_2	2_4	p-value	10_2	10_4	p-value
Bid1-Bid2	Vickrey	0.30	0.33	0.26	0.19	0.17	0.34	0.30	0.19	**0.00**	0.33	0.18	**0.00**
	Uniform	0.19	0.58	**0.00**	0.19	0.21	0.29	0.19	0.19	0.49	0.58	0.21	**0.00**
Bid2-Bid3	Vickrey	-	-	-	0.24	0.17	**0.05**	-	-	-	-	-	-
	Uniform	-	-	-	0.31	0.24	**0.03**	-	-	-	-	-	-
Bid3-Bid4	Vickrey	-	-	-	0.18	0.10	**0.00**	-	-	-	-	-	-
	Uniform	-	-	-	0.16	0.11	**0.03**	-	-	-	-	-	-

TABLE 8 - PROPORTION OF FLAT BID SCHEDULES (%)

Treatment	Flat bid_Bid1_Bid2		p-value	Flat bid_Bid2_Bid3		p-value	Flat bid_Bid3_Bid4		p-value
	Vickrey	Uniform		Vickrey	Uniform		Vickrey	Uniform	
10_2	17	12	0.32	-	-	-	-	-	-
10_4	30	20	0.10	27	25	0.75	49	48	0.89
2_2	13	14	0.84	-	-	-	-	-	-
2_4	29	25	0.52	23	18	0.38	28	45	**0.01**

TABEL 9 - THE EFFECT OF VARYING THE NUMBER OF BIDDERS AND THE NUMBER OF UNITS ON THE PROPORTION OF FLAT BID SCHEDULES

Flat bids		2_2	10_2	p-value	2_4	10_4	p-value	2_2	2_4	p-value	10_2	10_4	p-value
Flat bid_Bid1_Bid2	Vickrey	13	17	0.43	29	30	0.87	13	29	**0.00**	17	30	**0.03**
	Uniform	14	12	0.67	25	20	0.26	14	25	**0.05**	12	20	0.12
Flat bid_Bid2_Bid3	Vickrey	-	-	-	23	27	0.51	-	-	-	-	-	-
	Uniform	-	-	-	18	25	0.22	-	-	-	-	-	-
Flat bid_Bid3_Bid4	Vickrey	-	-	-	28	49	**0.00**	-	-	-	-	-	-
	Uniform	-	-	-	45	48	0.67	-	-	-	-	-	-

TABLE 10 - AVERAGE REVENUES

Treatment	Revenue		p-value
	Vickrey	Uniform	
10_2	4.01	3.96	0.44
10_4	4.36	4.16	0.20
2_2	1.16	1.03	0.14
2_4	1.61	1.53	0.35

TABLE 11- THE EFFECT OF VARYING THE NUMBER OF BIDDERS ON REVENUES

Auction format	2_2	10_2	p-value	2_4	10_4	p-value
Vickrey	1.16	4.01	**0.00**	1.61	4.36	**0.00**
Uniform	1.03	3.96	**0.00**	1.53	4.16	**0.00**

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